# Global Warming Potential, Optimal Abatement Targets, and the Design of Climate Policy

### Stéphane De Cara, Elodie Debove and Pierre-Alain Jayet

Institut National de la Recherche Agronomique (INRA), UMR Economie Publique INRA INA-PG, Grignon, France

This version: December 12, 2006

Abstract. We investigate the design of abatement targets in a multi-greenhouse gas and dynamic framework. Despite well-known shortcomings, the Global Warming Potential (GWP) is the most commonly used index to aggregate various greenhouse gases. We first characterize the 'first-best' abatement trajectories. We then turn to GWP-based targets, and focus on those that minimize the bias induced by the GWP. In such a second-best setting, we show that the optimal  $CO_2$ -equivalent target should be accommodated in order to account for the bias induced by the use of the GWP. Some implications for the design of climate policy are discussed.

JEL Codes: Q25

# Global Warming Potential, Optimal Abatement Targets, and the Design of Climate Policy

Abstract. We investigate the design of abatement targets in a multi-greenhouse gas and dynamic framework. Despite well-known shortcomings, the Global Warming Potential (GWP) is the most commonly used index to aggregate various greenhouse gases. We first characterize the 'first-best' abatement trajectories. We then turn to GWP-based targets, and focus on those that minimize the bias induced by the GWP. In such a second-best setting, we show that the optimal  $CO_2$ -equivalent target should be accommodated in order to account for the bias induced by the use of the GWP. Some implications for the design of climate policy are discussed.

**Keywords:** Global Warming Potential; climate change; climate policy; multigreenhouse gas agreements.

JEL codes: Q25.

## 1. Introduction

Few concepts derived from natural sciences have made their way into international law. The Global Warming Potential, or GWP for short, is one of them. In its Article 5.3, the Kyoto Protocol states that "the global warming potentials used to calculate the carbon dioxide equivalence of anthropogenic emissions [...] of greenhouse gases [...] shall be those accepted by the Intergovernmental Panel on Climate Change" (UNFCCC, 1997). The wording of the Kyoto Protocol therefore passes on the "legally-binding" nature of the Kyoto emission targets to the GWP concept itself.

As a matter of fact, the success of the GWP in the international negotiation arena may be seen as a failed attempt by economists to have sound economics translated into policy instruments. As soon as the early nineties, while the GWP concept was gaining momentum in both the scientific and policy debates, some of the most prominent economists in the field of climate change questioned its use for greenhouse gas (GHG) comparison purposes (Eckhaus, 1992; Schmalensee, 1993; Reilly and Richards, 1993). The concept was attacked on the grounds that it misleads the economically-sound choice of the mitigation mix. In other words, the GWP sets a "wrong currency" for comparing various GHGs. A dozen years of research later, one is left with the conclusion that the fundamental economic message contained in those criticisms was not successfully conveyed (Bradford, 2001).

The GWP has stood as a key feature in all assessment reports hitherto published by the IPCC (Houghton et al., 1990; 1995; 2001). Despite the caveats that were included in the latest IPCC Assessment Report (Ramaswamy et al., 2001), the importance of the GWP is not likely to fade away any time soon, and certainly not before the end of the first Kyoto commitment period. The concept is even commonly used by economists. Partly because of the status conferred by its inclusion in the Kyoto Protocol, and partly because the inherent economic inconsistencies it implies have been overlooked, the vast majority of economic assessments of the costs and/or the benefits of multi-gas mitigation strategies rely on the GWP concept. Yet the result that the GWP is ill-defined is a robust one from an economic standpoint. It has been confirmed by a number of studies, which, following up on the aforementioned pioneering works, proposed alternative indices (Kandlikar, 1995; Kandlikar, 1996; Hammit et al., 1996; Bradford and Keller, 2000; Manne and Richels, 2001; Shine et al., 2005) or provided empirical assessments of the concept's implications (Michaelis, 1999; Smith and Wigley, 2000; O'Neil, 2000; Tol et al., 2003; O'Neil, 2003; Kurosawa, 2004; Sarofim et al., 2005). See Fuglestvedt et al. (2003) for a comprehensive review.

Two strategies may then be envisaged to address multi-greenhouse gas issues. The continuation of frontal attacks to the GWP is one. This would involve continually trying to fit important economic concepts such as discounting, marginal abatement costs and marginal damage into the definition of a GHG index in the hope that this will eventually prove more successful than it has been in the last decade. An alternative—and more modest—approach focuses on second-best GWP-based economic instruments. This alternative approach recognizes that the GWP, albeit imperfect, must have compelling aspects that made it so successful as a policy concept. The challenge thus consists in designing economic instruments able to minimize the bias induced by the use of an imperfect metric. This latter approach is the one explored in the present paper.

The paper is structured as follows. In Section 2, we present the framework used to address multi-greenhouse issues and derive the analytical properties of optimal multi-GHG abatement paths. The bias induced by the use of GWP-based targets is illustrated in Section 3. Section 4 discusses the properties of second-best GWPbased multi-GHG targets. Section 5 examines the policy and economic implications of GWP-based instruments. Section 6 concludes.

## 2. Optimal multi-gas abatement path

In this section, we develop an analytical framework to investigate multi-greenhouse gas issues. The analytical properties of the first-best solution are first examined in order to use them later as a benchmark in the analysis of GWP-based instruments.

Consider a problem with n greenhouse gases indexed by  $j = 1, \ldots, n$ . The nvector<sup>1</sup> of atmospheric concentrations at time t is denoted by  $\mathbf{z}_t = t(z_{1t}, \ldots, z_{nt})$ . GHGs tend to decay in the atmosphere. Most of the literature that examines multigas issues from an analytical perspective assumes exponential decay processes characterized by constant decay rates (Moslener and Requate, 2005, for instance). This assumption has the advantage of greatly simplifying the computation of the optimal control problem by restricting it to a linear differential system. However, it overlooks two important features of the atmospheric behavior of GHGs. First, because of the complexity of the exchanges between different carbon reservoirs (atmosphere, ocean, terrestrial carbon pool) characterized by different transfer speeds between each of them, the carbon cycle can hardly be reduced to a simple, constant-rate decay process (Houghton et al., 2001). Second, interactions between the various GHGs in the atmosphere can significantly impact the speed at which they are decayed (Houghton et al., 2001). Therefore, we adopt here a fairly general formulation of the decay process. We represent this process by the n-vector valued function  $\mathbf{f}(\mathbf{z}_t) = {}^t(f_1(\mathbf{z}_t), \ldots, f_n(\mathbf{z}_t))$ . Each entry of  $\mathbf{f}()$ , denoted by  $f_j(\mathbf{z})$ , describes the decay process of gas j as a function of the full vector of concentrations.

Anthropogenic emissions in all GHGs are denoted by the *n*-vector  $\mathbf{e} = {}^{t}(e_{1t}, \ldots, e_{nt})$ and are measured in mass unit of each gas (tons of CO<sub>2</sub>, tons of methane, etc.). Net emissions are decomposed into two components: (i) business-as-usual emissions, which are denoted by  $\mathbf{\bar{e}}_{t} = {}^{t}(\bar{e}_{1t}, \ldots, \bar{e}_{nt})$ , and (ii) abatements, which are denoted by  $\mathbf{a}_{t} = {}^{t}(a_{1t}, \ldots, a_{nt})$ . The business-as-usual emission path is considered exogenous, and can be taken for instance from the IPCC scenarios (Nakicenovic and Swart, 2000).

<sup>&</sup>lt;sup>1</sup> Vectors and matrices are denoted in bold lower- and upper-case, respectively. All vectors are column vectors. The prescript <sup>t</sup> denotes the transposed operator.

The equation of motion over time of concentration is thus:

$$\dot{z}_{jt} = -f_j(z_{1t}, \dots, z_{nt}) + \bar{e}_{jt} - a_{jt}$$
 for  $j = 1, \dots, n$  (1)

or, in matrix form:

$$\dot{\mathbf{z}}_t = -\mathbf{f}(\mathbf{z}_t) + \bar{\mathbf{e}}_t - \mathbf{a}_t \tag{2}$$

The effect of GHGs on the climate is measured by  $\theta(\mathbf{z}_t)$ , which summarizes climate change through, for instance, the change in global mean surface temperature.  $\theta(.)$ depends on the full vector of concentrations, accommodating the possible countereffects that some gases can exert on the radiative impact of other gases. This is particularly important to account for the interactions between the radiative forcing of methane and that of nitrous oxide in the atmosphere (Ramaswamy et al., 2001).

In a welfare-based analysis of multi-GHG issues, climate impacts need to be translated into damages. Following a commonly used assumption in the literature, we consider that the economic loss due to climate change, denoted by  $D(\theta(\mathbf{z}_t))$ , is an increasing and convex function of the change in global surface temperature  $(D(0) = 0, D'(.) > 0, D''(.) \ge 0)$ . As we consider the optimal emission path from a global perspective, we focus on the global measure of the damage, not on spatially differentiated damages.

Abatement costs at time t, denoted by  $C(\mathbf{a}_t)$ , depend on the level of abatements in all GHGs. Again, a general formulation is important to account for potential interactions between the processes governing emissions, as well as between mitigation strategies. Agriculture provides an interesting illustration of such interactions. Mitigation strategies with respect to enteric fermentation (mostly methane) can take the form of reducing livestock numbers and/or modifying the way animals are fed. Both options have impacts on emissions from manure management (methane, but also nitrous oxide) and emissions from agricultural soils (mostly nitrous oxide). Separability of abatement costs—an assumption often retained in the literature between methane and nitrous oxide is thus hardly justified. In the general framework developed in this paper, we account for these interactions. However, for the sake of simplicity, we retain a quadratic<sup>2</sup> formulation:

$$C(a_{1t}, \dots, a_{nt}) = \frac{1}{2} \sum_{j=1}^{n} \sum_{k=1}^{n} c_{jk} a_{jt} a_{kt}$$
(3)

or, in matrix form:

$$C(\mathbf{a}_t) = \frac{1}{2} \, {}^t \mathbf{a}_t \cdot \mathbf{C} \cdot \mathbf{a}_t \tag{4}$$

That is, each entry of the vector of marginal abatement costs in all GHGs is assumed to be linear with respect to the vector of abatements ( $\mathbf{C} \cdot \mathbf{a}$ ). In order to fulfill the usual convexity requirements,  $\mathbf{C}$  is a  $n \times n$ , symmetric, and positive definite matrix. See Moslener and Requate (2005) for a discussion of the importance of non-diagonal entries in matrix  $\mathbf{C}$ . In addition, note that equation (4) implies that the abatement cost function is constant over time. That is, the formulation does not account for technical progress in the abatement technology, neither through an exogenous cost-decreasing trend nor a learning-by-doing process. Although this can arguably be important in deriving optimal targets, this would increase the complexity of the subsequent developments without fundamentally changing the nature of the results.

We now turn to the problem faced by a (risk-neutral) social planner, who seeks to set optimal abatement trajectories in *all* gases in order to minimize the sum of abatement costs and damage.  $\rho$  denotes the (constant) social discount rate. The corresponding program is:

$$\min_{\mathbf{a}_t} \int_0^{+\infty} \left[ C(\mathbf{a}_t) + D(\theta(\mathbf{z}_t)) \right] e^{-\rho t} dt \text{ subject to } (2)$$
(5)

The first-order optimality conditions for program (5) are given by<sup>3</sup>:

$$\mathbf{a}_{t}^{*} \in \arg\min_{\mathbf{a}_{t}} \mathcal{H}^{C} = C(\mathbf{a}_{t}) + D(\theta(\mathbf{z}_{t}^{*})) + {}^{t}\boldsymbol{\mu}_{t}^{*} \cdot (-\mathbf{f}(\mathbf{z}_{t}^{*}) + \bar{\mathbf{e}}_{t} - \mathbf{a}_{t})$$
(6a)

$$\dot{\mu}_{jt}^* = \rho \mu_{jt}^* - \frac{\partial \mathcal{H}^C}{\partial z_j} \text{ for all } j = 1, \dots, n$$
(6b)

<sup>&</sup>lt;sup>2</sup> The fact that marginal abatement costs in each gas are linear with respect to the whole vector of abatements **a** greatly simplifies the calculation of the abatement supply (equation (7)). More general assumptions on the abatement cost function are possible, provided that the Hessian matrix of  $C(\mathbf{a})$  is positive definite. This would require making use of the implicit function theorem and does not change the qualitative nature of the results.

<sup>&</sup>lt;sup>3</sup> The convention taken here for the sign of shadow prices implies that the public "bads" (concentrations of GHGs) are assigned a positive price.

where  $\mathcal{H}^C$  denotes the current Hamiltonian of program (5),  $\boldsymbol{\mu}_t$  denotes the *n*-vector of adjoint variables (or shadow prices) associated with the *n* equations of motion. Optimal levels of state, control and adjoint variables are denoted with a star.

The relationships (6a)–(6b), together with the equation of motion of  $\mathbf{z}_t$  (2), initial concentrations  $\mathbf{z}_0$ , and transversality conditions characterize the optimal abatement trajectories.

Equation (6a) implies that, at all point in time, optimal abatement in all gases should be such that marginal abatement in gas j is equal to the respective shadow price,  $\mu_{jt}^*$ . From equation (4), we can derive the optimal abatement supply in all gases as a function of the vector of shadow prices:

$$\mathbf{a}_t^* = \mathbf{C}^{-1} \cdot \boldsymbol{\mu}_t^* \tag{7}$$

Equations (6b) can be rewritten in matrix form as:

$$\dot{\boldsymbol{\mu}}_t^* = \left(\rho \mathbf{I}_n + \mathbf{J}_{\mathbf{f}}(\mathbf{z}_t^*)\right) \cdot \boldsymbol{\mu}_t^* - D'(\boldsymbol{\theta}(\mathbf{z}_t^*)) \mathbf{J}_{\boldsymbol{\theta}}(\mathbf{z}_t^*)$$
(8)

where  $\mathbf{I}_n$  is the  $n \times n$ -identity matrix,  $\mathbf{J}_{\mathbf{f}}(\mathbf{z}_t)$  is the  $n \times n$ -Jacobian matrix of  $\mathbf{f}(\mathbf{z}_t)$ , whose generic entry is defined as  $\frac{\partial f_j}{\partial z_k}(\mathbf{z}_t)$ , and  $\mathbf{J}_{\theta}(\mathbf{z}_t)$  is the  $n \times 1$ -Jacobian matrix of  $\theta(\mathbf{z}_t)$ , whose generic entry is  $\frac{\partial \theta}{\partial z_j}(\mathbf{z}_t)$ . The *j*-th row of the matrix  $\mathbf{J}_{\mathbf{f}}(\mathbf{z}_t)$  is thus the profile of the marginal impact of a change in the atmospheric composition on gas *j*'s concentration. Similarly, the *j*-th entry of  $\mathbf{J}_{\theta}(\mathbf{z}_t)$  reflects the marginal impact of gas *j* on global temperature.

Equation (8) imposes that each individual shadow price changes over time in such a way that it equals the present value of damage caused by a marginal increase in emissions in the respective gas. Introducing optimal abatements from equation (7) into equation (2) yields:

$$\dot{\mathbf{z}}_t^* = -\mathbf{f}(\mathbf{z}_t^*) + \bar{\mathbf{e}}_t - \mathbf{C}^{-1}\boldsymbol{\mu}_t^*$$
(9)

Together, equations (8) and (9) define a 2n first-order differential system in  $\mu_t$ and  $\mathbf{z}_t$ . Note that if D(.) is linear with respect to  $\theta$ , and  $\mathbf{f}(.)$  and  $\theta(.)$  are both linear with respect to  $\mathbf{z}_t$ , then equation (8) reduces to a linear first-order differential system with constant coefficients. The sub-system in  $\mu_t$  can then be solved independently of  $\mathbf{z}_t$ . In this case, the full system can be solved stepwise, solving first for  $\mu_t$ , then computing optimal abatement through equation (7), and finally solving for concentrations through equation (9). As soon as the non-linearities in either damage, climate responses, or atmospheric concentrations are taken into account, this simple step-wise solving method does not apply. Given our fairly general assumptions with respect to these variables, solving the full system is more of a numerical task than an analytical one.

#### 3. GWP-based abatement targets

Optimal responses to multi-dimensional issues generally require as many instruments as there are dimensions in the problem. Multi-GHG issues are not different. The previous section examined the solution of a *n*-dimensional problem (*n* GHGs) for which the social planner has *n* command variables at hand (*n* abatement paths). The use of the GWP, or of any constant metric in that regard, leaves the social planner with only one command variable by summarizing the full profile of emissions into one scalar (e.g., total  $CO_2$ -equivalent abatement). Yet, the problem is still a *n*dimensional one as soon as GHGs atmospheric behavior and climate impacts differ. By construction, GWP-based instruments are therefore likely to provide sub-optimal answers to multi-GHG issues. In this section, we examine the implications of GWPbased abatement targets.

Consider that a GWP-like GHG index has been agreed upon. This index allows converting emissions in any gas j into one particular reference gas. Without loss of generality, we assume that gas 1 is taken as the reference gas. For clarity of the exposition and in order to stay in line with the Kyoto Protocol's terminology, we refer to gas 1 as CO<sub>2</sub>. Let  $t\gamma = (1, \gamma_2, \ldots, \gamma_n)$  be the *n*-vector of conversion coefficients of gas j into CO<sub>2</sub>. As is the case for the GWPs, all entries of  $\gamma$  are assumed to be constant over time. Note that  $\gamma$  encompasses the standard definition of the GWP as a particular case, but also covers any kind of *constant* GHG index. As an illustration, CO<sub>2</sub>-only strategies can also be analyzed using this framework (in this case,  $\gamma_j = 0$  for all  $j \geq 2$ ). Total CO<sub>2</sub>-equivalent abatement at time t amounts to:

$$\sum_{j=1}^{n} \gamma_j a_{jt} = {}^t \boldsymbol{\gamma} \cdot \mathbf{a}_t \tag{10}$$

We proceed in two steps. First, we assume that a  $CO_2$ -equivalent target for the entire planning horizon has been set by the social planner. Agents adjust their abatement decisions in order to minimize the cost of meeting this target at all times. Second, the social planner chooses the optimal target knowing agents' responses.

Let us thus first assume that an aggregate target,  $\alpha_t$ , based on the  $\gamma$ -index, has been fixed. The corresponding cost minimization program is:

$$\min_{\mathbf{a}_t} C(\mathbf{a}_t) \text{ subject to } {}^t \boldsymbol{\gamma} \cdot \mathbf{a}_t \ge \alpha_t$$
(11)

which leads to the n following first-order conditions written in matrix form:

$$\mathbf{C} \cdot \mathbf{a}_t = \lambda_t \boldsymbol{\gamma} \tag{12}$$

The abatement profile at time t should be such that the marginal abatement cost in each gas is equal to the shadow price associated with the  $\gamma$ -aggregated target  $(\lambda_t)$  times the respective value of the GHG index. Note that at the optimum  $\lambda_t$ should thus be equal to the marginal abatement cost of CO<sub>2</sub>. If abatements are to be traded through a single emission permit system, the equilibrium price of gas j on this market should be equal to  $\gamma_j \lambda_t$ . This illustrates the fact that a multi-gas target sets the relative prices of the various GHGs.

By using the fact that the constraint in program (11) should be binding at the optimum, we can eliminate  $\lambda_t$ . The cost-minimizing abatement vector is denoted by a tilde and is obtained as a function of the CO<sub>2</sub>-equivalent target and  $\gamma$ :

$$\tilde{\mathbf{a}}(\alpha_t) = \frac{\alpha_t}{{}^t \boldsymbol{\gamma} \mathbf{C}^{-1} \boldsymbol{\gamma}} \mathbf{C}^{-1} \boldsymbol{\gamma}$$
(13)

By using equation (13), one can derive the total abatement cost implied by  $\alpha$  when used as the GWP-based target, and compare it to the abatement costs under the first-best regime:

$$C(\tilde{\mathbf{a}}(\alpha_t)) - C(\mathbf{a}^*) = \frac{1}{2} \frac{\alpha_t^2}{{}^t \boldsymbol{\gamma} \mathbf{C}^{-1} \boldsymbol{\gamma}} - \frac{1}{2} {}^t \boldsymbol{\mu}_t^* \mathbf{C}^{-1} \boldsymbol{\mu}_t^* = \frac{\left(\alpha_t^2 - {}^t \boldsymbol{\mu}_t^* \mathbf{C}^{-1} \boldsymbol{\mu}_t^* \cdot {}^t \boldsymbol{\gamma} \mathbf{C}^{-1} \boldsymbol{\gamma}\right)}{2 {}^t \boldsymbol{\gamma} \mathbf{C}^{-1} \boldsymbol{\gamma}}$$
(14)

Thus, a GWP-based abatement target induces lower abatement costs than the first-best optimal abatement path if (and only if):

$$\alpha_t \le \sqrt{{}^t \boldsymbol{\mu}_t^* \mathbf{C}^{-1} \boldsymbol{\mu}_t^* \cdot {}^t \boldsymbol{\gamma} \mathbf{C}^{-1} \boldsymbol{\gamma}}$$
(15)

If  $\alpha_t$  is such that inequality (15) is verified, the corresponding GWP-based abatement target allows saving on abatement costs. Therefore, for the first-best regime to dominate the GWP-based abatement regime at any time t, the difference in environmental damage between the two regimes has to exceed the RHS of equation (14).

Imagine now that the social planner uses the first-best abatement profile,  $\mathbf{a}_t^*$ , to compute  $\alpha_t$ . That is,  $\alpha_t$  is chosen to be equal to  $\alpha_t^* = {}^t \boldsymbol{\gamma} \cdot \mathbf{a}_t^*$ , with  $\mathbf{a}_t^*$  defined as in equation (7)<sup>4</sup>. Using equations (7) and (13), this yields:

$$\tilde{\mathbf{a}}(\alpha_t^*) = \frac{{}^t \boldsymbol{\gamma} \mathbf{C}^{-1} \boldsymbol{\mu}_t^*}{{}^t \boldsymbol{\gamma} \mathbf{C}^{-1} \boldsymbol{\gamma}} \mathbf{C}^{-1} \boldsymbol{\gamma}$$
(16)

The factor  $\frac{{}^{t} \gamma \mathbf{C}^{-1} \boldsymbol{\mu}_{t}^{*}}{{}^{t} \gamma \mathbf{C}^{-1} \gamma}$  appearing on the RHS of equation (16) measures the economic bias induced by the GWP. Given our assumptions on matrix  $\mathbf{C}$  (symmetric and positive definite), this factor can be geometrically interpreted as a particular measure of the angle between  $\gamma$  and  $\boldsymbol{\mu}^{*}$ . This measure uses the norm defined by  $\mathbf{C}^{-1}$ . In words, the measure of the bias weights the various GHGs according to their respective contribution to marginal abatement costs. Many papers that investigate GWP-related economic issues focus only on the ratio between the each gas' GWP and its respective shadow price (Kandlikar, 1996; Moslener and Requate, 2005; Börhinger et al., 2005). Equation (16) indicates that a comprehensive measure of the economic bias should account for the difference in the parameters defining the marginal abatement costs for each GHG<sup>5</sup>.

The comparison of the first-best abatement vector and the one resulting from the implementation of the GWP-based target  $\alpha^*$  yields:

$$\tilde{\mathbf{a}}(\alpha_t^*) - \mathbf{a}_t^* = \mathbf{C}^{-1} \left( \frac{{}^t \boldsymbol{\gamma} \mathbf{C}^{-1} \boldsymbol{\mu}_t^*}{{}^t \boldsymbol{\gamma} \mathbf{C}^{-1} \boldsymbol{\gamma}} \boldsymbol{\gamma} - \boldsymbol{\mu}_t^* \right)$$
(17)

 $<sup>^4</sup>$  Admittedly, one may find it odd that GWP-based targets are to be used whereas first-best abatements are assumed to be known. This indeed exemplifies the GWP paradox.

<sup>&</sup>lt;sup>5</sup> This result holds for more general formulation of the abatement cost function, provided that the Hessian matrix of abatement costs replaces  $\mathbf{C}$ .

The only solution for the full profile of abatement  $\tilde{\mathbf{a}}(\alpha_t^*)$  to be equal to the firstbest full profile of abatements is such that  $\gamma = k \boldsymbol{\mu}_t^*$  where k is any positive real scalar. Given the additional convention that  $\gamma_1 = 1$ , the only solution for  $\tilde{\mathbf{a}}(\alpha_t^*) = \mathbf{a}_t^*$  to hold is that  $\gamma_j = \frac{\mu_{jt}^*}{\mu_{1t}^*}$  for all  $j = 1, \ldots, n$ , and at all time. This result embeds the essence of the critical views of the GWP concept and illustrates the fundamental economic result with regard to the GHG equivalence factor. The (first-best) equivalence rule should be based on the shadow prices ratios.

For  $\alpha_t = \alpha_t^* = {}^t \boldsymbol{\gamma} \cdot \mathbf{a}_t^*$ , the GWP-based abatement target allows saving on abatement costs compared to the first-best abatement path at all time. This is readily verified by noticing that inequality (15) holds in this case as a direct application of the Cauchy-Schwarz inequality. By construction,  $\mathbf{a}^*$  minimizes the net present value of the sum of abatement costs and environmental damage. Consequently, the net present value of environmental damage is larger under a GWP-based abatement target regime than under the first-best regime, although the aggregate abatement when expressed in tons of CO<sub>2</sub>-equivalent— is identical under both regimes and equal to  $\alpha^*$ . The increase in the net present value of damage more than offsets the net present value of the savings in abatement costs.

## 4. Second-best, GWP-based abatement targets

The next step in our analysis consists of setting the best possible  $CO_2$ -equivalent target from a social welfare point of view. We know that GWP-based quantity instruments lead to a distortion in the abatement mix. The question is then: Is it possible to reduce this distortion? This section examines a class of *second-best* GWP-based instruments. The corresponding social planner's problem is:

$$\min_{\alpha_t} \int_0^{+\infty} \left[ C(\mathbf{a}_t) + D(\theta(\mathbf{z}_t)) \right] e^{-\rho t} dt \text{ subject to } (2) \text{ and } \mathbf{a}_t = \tilde{\mathbf{a}}(\alpha_t)$$
(18)

For any level of the CO<sub>2</sub>-equivalent target  $\alpha_t$ , abatements are supplied according to equation (13). Introducing (13) in the objective function and in the equation of motion of  $\mathbf{z}_t$ , one can form the current Hamiltonian  $\hat{\mathcal{H}}^C$  and derive the following first-order optimality conditions (optimal values are now signaled with a hat):

$$\hat{\alpha}_{t} \in \arg\min_{\alpha_{t}} \hat{\mathcal{H}}^{C} = \frac{\alpha_{t}^{2}}{2 t_{\gamma} \mathbf{C}^{-1} \gamma} + D(\theta(\hat{\mathbf{z}}_{t})) + t_{\mu} \left( \bar{\mathbf{e}}_{t} - \mathbf{f}(\hat{\mathbf{z}}_{t}) - \frac{\alpha_{t} \mathbf{C}^{-1} \gamma}{t_{\gamma} \mathbf{C}^{-1} \gamma} \right) (19a)$$
$$\dot{\hat{\boldsymbol{\mu}}}_{t} = \left( \rho \mathbf{I}_{n} + \mathbf{J}_{\mathbf{f}}(\hat{\mathbf{z}}_{t}) \right) \cdot \hat{\boldsymbol{\mu}}_{t} - D'(\theta(\hat{\mathbf{z}}_{t})) \mathbf{J}_{\theta}(\hat{\mathbf{z}}_{t})$$
(19b)

Solving problem (19a) for  $\hat{\alpha}_t$  yields:

$$\hat{\alpha}_t = {}^t \boldsymbol{\gamma} \mathbf{C}^{-1} \hat{\boldsymbol{\mu}}_t \tag{20}$$

The second-best CO<sub>2</sub>-equivalent target thus depends on the full vector of shadow prices derived from program (18). From equation (19b), one sees that  $\hat{\mu}_t$  is different from  $\mu_t^*$  as soon as current concentrations  $\hat{\mathbf{z}}_t$  differ from the first-best levels  $\mathbf{z}_t^*$ , that is, if either D(.),  $\theta(.)$  and/or  $\mathbf{f}(.)$  is non-linear. Introducing the expression of  $\hat{\alpha}_t$  in equation (13) gives the second-best vector of abatements:

$$\hat{\mathbf{a}}_t = \frac{{}^t \boldsymbol{\gamma} \mathbf{C}^{-1} \hat{\boldsymbol{\mu}}_t}{{}^t \boldsymbol{\gamma} \mathbf{C}^{-1} \boldsymbol{\gamma}} \mathbf{C}^{-1} \boldsymbol{\gamma}$$
(21)

which leads to the following equation of motion for  $\hat{\mathbf{z}}_t$ :

$$\dot{\hat{\mathbf{z}}}_t = -\mathbf{f}(\hat{\mathbf{z}}_t) + \bar{\mathbf{e}}_t - \frac{{}^t \boldsymbol{\gamma} \mathbf{C}^{-1} \hat{\boldsymbol{\mu}}_t}{{}^t \boldsymbol{\gamma} \mathbf{C}^{-1} \boldsymbol{\gamma}} \mathbf{C}^{-1} \boldsymbol{\gamma}$$
(22)

Abatement costs under the GWP-based abatement target regime are increasing with respect to  $\alpha$ . Hence, abatement costs under the second-best regime (CO<sub>2</sub>equivalent target  $\hat{\alpha}_t$ ) are greater than those under the GWP-based target ( $\tilde{\mathbf{a}}(\alpha_t^*)$ ) if and only if  $\hat{\alpha}$  is greater than  $\alpha^*$ ; that is, if and only if  $\boldsymbol{\mu}_t^*$  and  $\hat{\boldsymbol{\mu}}_t$  are such that  ${}^t\boldsymbol{\gamma}\mathbf{C}^{-1}\hat{\boldsymbol{\mu}}_t \geq {}^t\boldsymbol{\gamma}\mathbf{C}^{-1}\boldsymbol{\mu}_t^*$ . As readily seen from equation (14), the second-best abatement path induces greater abatement costs than under the first-best regime if and only if  $\boldsymbol{\mu}_t^*$  and  $\hat{\boldsymbol{\mu}}_t$  are such that  ${}^t\boldsymbol{\gamma}\mathbf{C}^{-1}\hat{\boldsymbol{\mu}}_t \geq \sqrt{{}^t\boldsymbol{\mu}_t^*\mathbf{C}^{-1}\boldsymbol{\mu}_t^*}$ .  ${}^t\boldsymbol{\gamma}\mathbf{C}^{-1}\boldsymbol{\gamma}$ . The ranking of abatement costs under the three regimes is summarized in Figure 1. In the first two cases depicted in Figure 1, abatement costs are the greatest under the first-best regime. As  $\mathbf{a}_t^*$  minimizes the net present value of abatement costs and damage together, one can conclude that, in this case, the net present value of damage is higher under the second-best regime than under the first-best regimes. In words, the second-best CO<sub>2</sub>-equivalent target, if not too high, allows for some saving on abatement cost, but leads to higher environmental damage. In the third case depicted in Figure 1, the second-best  $CO_2$  abatement target is sufficiently large compared to the  $CO_2$ -equivalence of the first-best abatement so as to induce larger abatement costs. The effect on damage is therefore ambiguous.

$$\alpha_t^* = {}^t \boldsymbol{\gamma} \mathbf{C}^{-1} \boldsymbol{\mu}_t^* \qquad \sqrt{{}^t \boldsymbol{\mu}_t^* \mathbf{C}^{-1} \boldsymbol{\mu}_t^* \cdot {}^t \boldsymbol{\gamma} \mathbf{C}^{-1} \boldsymbol{\gamma}} \qquad \hat{\alpha}_t = {}^t \boldsymbol{\gamma} \mathbf{C}^{-1} \hat{\boldsymbol{\mu}}_t$$

$$C(\hat{\mathbf{a}}_t) \leq C(\hat{\mathbf{a}}(\alpha_t^*)) < C(\mathbf{a}_t^*)$$

$$C(\hat{\mathbf{a}}(\alpha_t^*)) < C(\hat{\mathbf{a}}_t) \leq C(\hat{\mathbf{a}}_t)$$

$$C(\hat{\mathbf{a}}(\alpha_t^*)) < C(\hat{\mathbf{a}}_t) \leq C(\hat{\mathbf{a}}_t)$$

Figure 1. Ranking of abatement costs under GWP-based abatement target  $(\tilde{\mathbf{a}}(\alpha_t^*))$ , second-best GWP-based abatement target  $(\hat{\mathbf{a}}_t)$ , and first-best abatement target  $(\mathbf{a}_t^*)$ 

# 5. Discussion: Policy and economic implications

Flexibility is put forward as a key component of a successful, cost-effective climate policy architecture. Flexibility is commonly categorized into "where"-, "when"-, and "what"-flexibility. The debate over any GHG equivalence rule underlines the importance of GHG trade-offs and, therefore, is logically linked to the "what"-flexibility issue (Börhinger et al., 2005). The estimated cost-savings permitted by "what"flexibility are high, especially when one compares the costs associated to  $CO_2$ -only strategies with that of multi-GHG mitigation strategies (Reilly et al., 1999; Hayhoe et al., 1999). The magnitude of expected cost-savings related to additional "what"-flexibility certainly played a role in the success of the GWP concept.

Implications of the issues that are raised by the GWP however go beyond "what"flexibility. They cannot be disconnected from the analysis of "where"- and "when"flexibility. First, at the core of the critical views of the GWP concept lies the tradeoff between short- and long-lived GHGs (Aaheim, 1999; Michaelowa, 2003). The definition of any equivalence rule between various GHGs is thus crucial for the *timing* of mitigation strategies. Second, the GWP debate is also strongly linked to "where"flexibility in both a sectoral and geographical sense. The relative contribution of non $CO_2$  emissions varies widely across sectors and countries. In this regard, agriculture provides a good illustration. The contribution of this sector, which is the major emitting sector for non-CO<sub>2</sub> GHGs, to global reductions in GHG emissions heavily depends on the value attached to methane and nitrous oxide abatements relatively to  $CO_2$  abatements. Given the importance of agriculture in developing countries' economies and the high share of non-CO<sub>2</sub> emissions from rice cultivation (methane), nitrogen fertilization (nitrous oxide), and livestock production (methane and nitrous oxide), multi-gas targets and the relative weights attached to non-CO<sub>2</sub> gases may play a crucial role in getting developing countries on board in a post-Kyoto world. The stakes are thus high with regard to the design of a multi-gas climate policy architecture that is both negotiable and economically-sound.

Non-CO<sub>2</sub> GHGs are thus important in several respects. In the short run, they are likely to play a key role in closing the gap between the Kyoto targets and rising CO<sub>2</sub> emission trends. The fact that a large share of the Clean Development Mechanism projects registered by the UNFCCC<sup>6</sup> concentrate on non-CO<sub>2</sub> emissions provides a good illustration in this regard. In the longer run, they can contribute to emission targets that have a significant impact on climate change and, at the same time, broaden the set of participating countries in a post-Kyoto architecture.

The multi-gas issue can theoretically be solved by either setting targets for each individual GHG, or alternatively, assigning each GHG a price that adequately reflects the marginal abatement cost and the flow of future marginal damage. In the former case, individual GHG trading systems could be established to achieve costefficiency. In the latter case, trading could be done in a single CO<sub>2</sub>-equivalent market, the exchange rates between GHGs being set using the shadow prices calibrated on existing knowledge of climate/economic relationships. The choice between price and quantity instruments would then rely on the discussion of the uncertainty affecting damage and costs. In both cases, instruments (either targets or prices) need to change over time to account for the changes in atmospheric concentrations, and therefore in damage.

<sup>&</sup>lt;sup>6</sup> See http://cdm.unfccc.int/Projects/registered.html.

If, for some reason, the GWP still remains a cornerstone of the future design of climate policy, economic instruments have to be adapted to account for the resulting bias. That means, as shown in the previous section, that GWP-based targets may have to be more stringent than the  $CO_2$ -equivalence of the abatement path prescribed by integrated assessment models. The very fact that policymakers are left with only one command variable to cope with a multi-dimensional issue requires overshooting in order to offset (some of) the bias induced by the GWP.

The difference between first-best and second-best targets, when both are expressed in terms of GWP-based  $CO_2$ -equivalent, is shown as depending on two factors. First, this difference depends on how the equivalence factors reflect the shadow prices of each respective GHG. The GWP, by construction, is not likely to be the right candidate to fulfill this objective. Even if, by mere chance, the vector of GWPs provided a good proxy for the vectors of the shadow prices at some point in time, this would not hold over time because shadow prices are subject to change as concentrations and damage change.

Second, the difference between first- and second-best  $CO_2$ -equivalent targets depends on the gap between shadow prices under first- and second-best regimes. This, in turn, depends on the difference between first- and second-best concentration, temperature, and damage paths. For short-run targets, which correspond to small changes in concentrations, linear approximation might be adequate. In this case, the difference between first- and second-best shadow prices remains small, and so is the difference between first- and second-best  $CO_2$ -equivalent targets. But, if GWPs are to be used for a longer time period and/or the dynamics of the system (either in concentrations, temperature, or damage) is characterized by strong non-linearities, this gap has to rise in order to accommodate the increasing discrepancy between first- and second-best shadow prices.

An important policy implication is that second-best GWP-based targets have to be updated on a regular basis in order to account for the change in concentrations and damage. To determine how often the targets should be revised, one has to weight the transaction costs associated with negotiating them and the welfare impacts. Non-CO<sub>2</sub> abatement costs play a key role. The importance given to one particular gas cannot be reduced to its impact on climate but should also weigh the costs at which abatements can be supplied. As it is, the GWP does not reflect these differences. By contrast, the second-best instruments examined in the previous section do account for the differences in abatement costs across gases, as marginal abatement costs enter the calculation of abatement targets.

Interestingly, the establishment of second-best GWP-based instruments would also force clarifying the assumptions taken with respect to discount rate and damage, as these two elements determine the calculation of the target. Reaching an agreement on these assumptions is arguably challenging. However, economists could make the case that clear and consistent assumptions in this regard are in any respect better than the simplistic ones implied by the use of GWP.

## 6. Concluding remarks

The paces of policymaking and research are seldom synchronized. Often, scientists have to wait years or decades before they can see their concepts and results translated into policy. And sometimes policy moves ought to be made before science has had time to establish the necessary results. Global warming is one of the few examples of intense dialogue between interdisciplinary research and policymaking. The adoption of the GWP as the GHG "currency", however, stands out as a contrasting failure in this overall successful picture.

Is that to say that economists should throw in the towel in the event that the GWP is preferred over economic indices in the future design of climate policy? The analysis conducted in this paper shows that some room exists to adapt GWP-based targets in order to correct some of the distortions caused by the GWP. The role of economists would then be to emphasize that a CO<sub>2</sub>-equivalent target have to differ from what would be necessary with the right "currency".

How large the difference between first- and second-best targets should be is very much an empirical question. Our analytical results show that this depends on the difference between the GWP and the shadow prices, the magnitude of marginal abatement costs, and the (non)-linearities in both economic and ecological systems. Further research is needed to assess this gap based on state-of-the-art integrated modelling approach.

# References

- Aaheim, H. A.: 1999, 'Climate policy with multiple sources and sinks of greenhouse gases'. Environmental and Resource Economics 14(3), 413–429.
- Bradford, D. F.: 2001, 'Global change: Time, money and tradeoffs'. *Nature* **410**(6829), 649–650.
- Bradford, D. F. and K. Keller: 2000, 'Global Warming Potentials: A Cost-Effectiveness Approach'. Mimeo, Princeton University, Cambridge, MA, USA.
- Börhinger, C., A. Löschel, and T. F. Rutherford: 2005, 'Efficiency gains from "What"flexibility in climate policy: An integrated CGE assessment'. In: Venice Summer Institute: David F. Bradford Memorial Conference on The Design of Climate Policy. Venice, Italy. Selected paper.
- Eckhaus, R. S.: 1992, 'Comparing the effects of greenhouse gas emissions on global warming'. The Energy Journal 13(1), 25–35.
- Fuglestvedt, J. S., T. K. Berntsen, O. Godal, R. Sausen, K. P. Shine, and T. Skodvin: 2003, 'Metrics of climate change: Assessing radiative forcing and emission indices'. *Climatic Change* 58(3), 267–331.
- Hammit, J. K., A. Jain, J. L. Adams, and D. Wuebbles: 1996, 'A welfare-based index for assessing environmental effects of greenhouse-gas emissions'. *Nature* 381, 301–303.
- Hayhoe, K., A. Jain, H. Pitcher, C. MacCracken, M. Gibbs, D. Wuebbles, R. Harvey, and D. Kruger: 1999, 'Costs of multigreenhouse gas reduction targets for the USA'. Science 286, 905–906.
- Houghton, J., Y. Ding, D. Griggs, M. Noguer, P. van der Linden, X. Dai, K. Maskell, and C. Johnson (eds.): 2001, *Climate Change 2001: The Scientific Basis*, Vol. I of

*IPCC Third Assessment Report.* Cambridge, UK: Cambridge University Press. 881 p.

- Houghton, J., G. J. Jenkins, and J. J. Ephraums (eds.): 1990, Scientific Assessment of Climate change – Report of Working Group I, Vol. I of IPCC First Assessment Report. Cambridge, UK: Cambridge University Press. 365 p.
- Houghton, J., L. Meira Filho, B. Callander, N. Harris, A. Kattenberg, and K. Maskell (eds.): 1995, Climate Change 1995: The Science of Climate Change, Vol. I of IPCC Second Assessment Report. Cambridge, UK: Cambridge University Press. 572 p.
- Kandlikar, M.: 1995, 'The relative role of trace gas emissions in greenhouse abatement policies'. *Energy Policy* 23(10), 879–883.
- Kandlikar, M.: 1996, 'Indices for comparing greenhouse gas emissions: integrating science and economics'. *Energy Economics* 18, 265–281.
- Kurosawa, A.: 2004, 'Multigas reduction strategy under climate stabilization target'. Contributed Paper 181, 7th International Conference on Greenhouse Gas Control Technologies, Vancouver, Canada.
- Manne, A. S. and R. G. Richels: 2001, 'An alternative approach to establishing trade-offs among greenhouse gases'. Nature 410, 675 – 677.
- Michaelis, P.: 1999, 'Sustainable greenhouse policies: the role of non-CO<sub>2</sub> gases'. Structural Change and Economic Dynamics 10, 239–260.
- Michaelowa, A.: 2003, 'Limiting global cooling after global warming is over Differentiating between short- and long-lived greenhouse gases'. OPEC Review 27(4), 343–351.
- Moslener, U. and T. Requate: 2005, 'Optimal abatement in dynamic multi-pollutant problems when pollutants can be complements or substitutes'. Economics Working Paper 2005-03, Christian-Albrechts-Universitaet Kiel, Department of Economics, Kiel, Germany.
- Nakicenovic, N. and R. Swart (eds.): 2000, Special Report on Emissions Scenarios. Cambridge, UK: Cambridge University Press. 612 pp.
- O'Neil, B. C.: 2000, 'The jury is still out on Global Warming Potentials. An editorial comment'. *Climatic Change* 44(4), 427–443.

- O'Neil, B. C.: 2003, 'Economics, natural science, and the costs of Global Warming Potentials. An editorial comment.'. *Climatic Change* 58(3), 251–260.
- Ramaswamy, V., O. Boucher, J. Haigh, D. Hauglustaine, J. Haywood, G. Myhre, T. Nakajima, G. Shi, and S. Solomon: 2001, *Radiative Forcing of Climate Change*, Chapt. 6, pp. 349–416. Vol. I of (Houghton et al., 2001). 881 p.
- Reilly, J. M., R. G. Prinn, J. Harnisch, J. Fitzmaurice, H. D. Jacoby, D. Kicklighter, J. Melillo, P. Stone, A. Sokolov, and C. Wang: 1999, 'Multi-gas assessment of the Kyoto Protocol'. *Nature* 401, 549–555.
- Reilly, J. M. and K. H. Richards: 1993, 'Climate change damage and the trace gas index issue'. *Environmental and Resource Economics* 3, 41–61.
- Sarofim, M. C., C. E. Forest, D. M. Reiner, and J. M. Reilly: 2005, 'Stabilization and global climate policy'. *Global and Planetary Change* 47(2–4), 266–272.
- Schmalensee, R.: 1993, 'Comparing greenhouse gases for policy purposes'. The Energy Journal 14(1), 245–256.
- Shine, K. P., J. S. Fuglestvedt, K. Hailemariam, and N. Stuber: 2005, 'Alternatives to the Global Warming Potential for comparing climate impacts of emissions of greenhouse gases'. *Climatic Change* 68(3), 281–302.
- Smith, S. J. and T. M. L. Wigley: 2000, 'Global Warming Potentials: 1. Climatic implications of emissions reductions'. *Climatic Change* 44(4), 445–457.
- Tol, R. S. J., R. J. Heintz, and P. E. M. Lammers: 2003, 'Methane emission reduction: An application of FUND'. *Climatic Change* 57(1-2), 71–98.
- UNFCCC: 1997, 'Kyoto Protocol to the United Nation Framework Convention on Climate Change'. Cop3, UNFCC (Climate Change Secretariat).