Multi-Greenhouse gas International Agreements

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Abstract:

We examine the influence of the multi-pollutant nature of climate change on the stability of international environmental agreements. We consider a $n$-player, two-stage game. Pollution results from two gases, which differ by their environmental impact and their abatement costs. Different kinds of agreements are examined: 'single-gas agreements' (one pollutant is neglected in international negotiations), 'comprehensive agreements' (the treaty encompasses the two pollutants), and 'gas-by-gas agreements' (the abatements in the two pollutants are set up separately in two different agreements). In each case, the outcome of the emission game is computed for any given partition of the set of countries. The stability conditions are used to determine the outcome of the first-stage game. In the case of homogeneous countries, we show that the size of stable agreements remains low (no more than two countries). We give necessary and sufficient conditions of existence of such agreements. Our main result shows that comprehensive agreements better resist free-riding incentives and thus are more likely to emerge than single-gas agreements.

Keywords:

Self-enforcing international environmental agreements, multiple pollutants, greenhouse gases, climate change, Kyoto Protocol

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Accords internationaux sur les changements climatiques :
Influence stratégique de la présence de plusieurs polluants

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Résumé :

Nous examinons l'influence de la présence de plusieurs polluants sur la stabilité des accords environnementaux internationaux. Pour ce faire, nous étudions un jeu en deux étapes à \( n \) joueurs. La pollution est le résultat de l'action combinée de deux polluants, qui diffèrent par leur impact sur l'environnement et les coûts d'abattement qui leur sont associés. Différents types d'accords sont examinés : les accords 'simples' (seul un polluant est considéré dans les négociations), les accords 'complets' (l'ensemble des polluants est pris en compte dans les négociations) et les accords 'gaz par gaz' (des accords différents pour chaque gaz co-existent). Dans chaque cas nous caractérisons complètement l'issue du jeu en émissions pour une composition donnée de l'accord environnemental. Les conditions de stabilité interne et externe sont ensuite utilisées pour résoudre le jeu d'adhésion. Dans le cas de pays homogènes, nous montrons que la taille des accords environnementaux demeure faible (pas plus de deux pays). Une des originalités de nos résultats est la caractérisation complète de l'existence d'accords stables pour des formes de coûts et de bénéfice environnementaux largement utilisées dans la littérature. Ces conditions nous permettent de montrer que les accords complets résistent mieux aux incitations à se comporter en passager clandestin.

Mots Clés :

Accords internationaux environnementaux ; Multi-gaz ; Stabilité ; Gaz à effet de serre ; Changements climatiques ; Accord de Kyoto

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Abbreviations: COP – Conference of the Parties; GWP – Global Warming Potential; IPCC – Intergovernmental Panel on Climate Change; PANE wrt $S$ – Partial Agreement Nash Equilibrium with respect to coalition $S$; UNFCCC – United Nations Framework Convention on Climate Change

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Introduction

International environmental issues are still characterized by the lack of a real supranational body able to enforce Pareto-improving measures. As a consequence, any action should arise from an intergovernmental negotiation process. In the case of climate change, the public-bad nature of greenhouse gases emissions also implies that such decisions may be undermined by free-riding incentives. The reversal in the US policy regarding climate change highlights how difficult it can be to design efficient and effective international environmental agreements. So far, more than ten years after the Rio Convention and despite climate scientists’
growing alarm (Intergovernmental Panel on Climate Change, 2001), international action is still merely restricted to a “global warning”.

During the last decade, an important body of literature has addressed these issues from a game-theoretic perspective. Carraro and Siniscalco (1992) first argued that the “tragedy of the commons” could be somehow overcome thanks to a partial cooperation among countries. Their approach based on the use of cartel stability concepts shows that, even with a lack of international government, self-enforcing agreements may emerge as a stable equilibrium of a non-cooperative game. Extensive surveys of the related literature can be found in Barrett (1997b), Carraro (1998) and Finus (2000). Nevertheless, these studies show that the size of such self-enforcing agreements remains generally low in terms of number of signatories. Therefore, such agreements are far from achieving the cooperative outcome.

In Kyoto, extending the set of gases in the targets definition from three to six gases has captured a lot of negotiators’ efforts to reach an agreement. Likewise, deciding whether to include or not carbon sinks in emission targets has been one of the key-issues in the negotiation. Surprisingly, whereas the definition of the “basket of pollutants” to be included in the Kyoto Protocol has been at core of the recent talks, it has received little attention in economics studies, which analyze strategic dimensions of climate change negotiations. As stated by Hoel (1991): “Whatever type of international agreement is reached during the next decade, it will probably only cover CO$_2$ – not other climate gases. [...] Although agreements encompassing all climate gases could be more efficient, practical considerations, will thus force governments, at least initially, to limit an agreement to CO$_2$."

In the immediate aftermath of the Rio Conference, evidence from the negotiation process appears to be in contradiction with Hoel’s prediction. Admittedly, as suggested by Hoel, the importance of energy-related emissions and their straightforward link with economic growth may ease the monitoring of CO$_2$ emissions. But indeed, once quantitative targets are adopted, the wider the ”portfolio” of pollutants is, the larger the opportunity is to find low-cost abatement options for a given level of the emissions target (Manne and Richels, 2000).

Recent estimates unambiguously show the cost-effectiveness of a multi-gas approach (Hayhoe et al., 1999; Reilly et al., 1999; Burniaux, 2000). This issue is also of great importance insofar as the choice of the set of pollutants strongly determines the sectors that should be involved in a regulation policy. This is particularly true for agriculture that accounts for most of the world emissions of nitrous oxide and methane and provides carbon storage both in soils and trees (Babcock and Pautsch, 1999; De Cara and Jayet, 2000; Schneider, 2000).
Both global environmental results and abatement costs are sensitive to the definition of the basket of pollutants. So must be the incentives to participate to an international agreement. The question examined in this paper is then to assess the influence of the set of pollutants on the formation of stable and self-enforcing agreements. We analyze the formation of international environmental agreements as the equilibrium of a two-stage non-cooperative game.

The model, the assumptions, and the different kinds of agreement we consider are defined in section 1. Section 2 is devoted to the analysis of the 'single-gas agreement' game, in which only one gas is included in the treaty, other possible sources of pollution being ignored. The outcome of the emission game is computed for any given size of the agreement. The use of the stability conditions makes the size of the agreement endogenous. It is shown that only low-sized agreements (two countries) are likely to emerge. For a commonly used class of payoff functions, we give necessary and sufficient conditions of the existence of a stable agreement. These results are then extended to ‘comprehensive agreements’, which encompass all the pollutants (section 3). ‘Gas-by-gas agreements’ are then considered as a generalization of the two former kinds of agreement in section 4. In this section, the environmental and economic results of each kind of agreement are compared and discussed. It is shown that comprehensive agreements not only lower abatement costs, but also better resist to free-riding incentives.

1. The model

Let consider a problem, in which \( n \) countries share a common resource, namely the atmosphere or the climate. The set of the \( n \) countries is denoted by \( I = \{1, \ldots, n\} \).

1.1. One environmental issue, two pollutants

The pollution is assumed to result from two gases, denoted by \( g_1 \) and \( g_2 \). The abatement in gas \( g_j \), \( j \in J = \{1, 2\} \) in country \( i \in I \) is denoted by \( q_{ij} \). These gases are considered to differ by their impact on the environment. An equivalence rule is assumed to have been established by the scientists and to have been accepted by the parties before the negotiations. As it is the case for GWP (for a given time horizon\(^4\)), the equivalence rule between the two gases is simply defined as a constant factor of conversion that allows to convert emissions of \( g_2 \) into \( g_1 \) equivalent. Hence, one mass unit of \( g_2 \) emitted now is assumed to be equivalent to \( \beta \) units of \( g_1 \) emitted now.\(^5\)
The total reduction in pollution is denoted by $Q$ and is computed as follows:

$$Q = \sum_{i \in I} (q_{i1} + \beta q_{i2}) = \sum_{i \in I} q_i$$

where $q_i$ is the national total abatement in country $i$, expressed in terms of $g_1$-equivalent.

### 1.2. Net benefits

Each country benefits from global reductions in the ambient pollution level. Following Barrett’s assumption (1994), the gross benefit function is assumed to be increasing and concave in $Q$ such that:

$$B_i(Q) = \frac{b_i}{n} \left(aQ - \frac{1}{2}Q^2\right) \quad \text{with} \quad b_i, a > 0 \quad \forall i \in I \quad (1)$$

Abatement costs faced by each country depend uniquely on its own abatement and are increasing and convex in $q_{ij}$. We express the function of abatement costs in terms of $g_j$ quantities so that:

$$C_{ij}(q_{ij}) = \frac{1}{2}c_{ij}q_{ij}^2 \quad \text{with} \quad c_{ij} > 0 \quad \forall (i, j) \in I \times \{1, 2\} \quad (2)$$

Net benefit of country $i$ thus depends on the vector of abatement decisions:

$$\pi_i(q_{ij}, q_{-i,j}) = B_i(Q) - C_{i1}(q_{i1}) - C_{i2}(q_{i2})$$

$$= \frac{b_i}{n} \left(aQ - \frac{1}{2}Q^2\right) - \frac{1}{2}c_{i1}q_{i1}^2 - \frac{1}{2}c_{i2}q_{i2}^2 \quad (3)$$

### 1.3. International environmental agreement(s) as a two-stage game

The generic game is a two-stage game. Countries decide non-cooperatively to join or not to join the environmental 'coalition(s)' in the first stage. This choice is assumed to be driven by the maximization of each country’s own individual net benefit. The first stage game is a simultaneous open-membership game. The second-stage game is the emission game. At this stage, countries set up their abatement levels. This choice is made jointly among the countries belonging to the agreement(s) and non-cooperatively for other countries. The partition of $I$ is assumed to be common knowledge as decided in the first stage.

One important feature of the formulation (3) is that the best-reply functions are non-orthogonal because of the quadratic setting of the
benefit function. As noted by Botteon and Carraro (1997), this implies the presence of some leakage, due to the possibility for non-signatory countries to increase their emissions in reaction to signatories’ decisions. This leakage tends to limit the size of stable agreements.\(^7\)

1.3.1. Second stage: Emission game
Following the above quoted Hoel’s assertion, single-gas agreements are first considered. In this case, countries are assumed to only consider one gas in the negotiation process. This kind of agreement thus leads to partition the set \(I\) into two types of countries: those which sign the environmental treaty on gas \(g_j\) and those which do not. For an exogenous reason, gas \(g_{-j}\) is not considered and abatement in this gas is set up individually by each country.\(^8\)

**DEFINITION 1 (Single-gas \(j\)-agreement).** A single-gas \(j\)-agreement is given by the partition of \(P(I) = \{S_j, \{i\}_{i \in I \setminus S_j}\}\) such that countries belonging to \(S_j\) choose cooperatively their abatement in gas \(g_j\), while other countries behave like singletons. The abatement levels in gas \(g_{-j}\) are set up non-cooperatively by all the countries. These decisions are assumed to occur simultaneously.

The underlying concept of equilibrium that is used here is a Partial Agreement Nash Equilibrium with respect to a coalition \(S_j\) (PANE wrt \(S_j\)).\(^9\) The equilibrium of the second-stage game is given by solving the following problem:

\[
P^{SG_j} \begin{cases} 
\max_{(q_i)_{i \in S_j}} \sum_{k \in S_j} \pi_k(q_k, q_{-k}) \\
\max_{q_{i,j}} \pi_i(q_i, q_{-i}) & \forall i \in I \setminus S_j \\
\max_{q_{i,-j}} \pi_i(q_i, q_{-i}) & \forall i \in I 
\end{cases}
\]

which first-order conditions are given by the following 2\(n\)-system:\(^{10}\)

\[
\sum_{k \in S_1} B'_{ki}(Q) = C'_{i1}(q_i) & \forall i \in S_1 \\
B'_{i1}(Q) = C'_{i11}(q_i) & \forall i \in I \setminus S_1 \\
\beta B'_{i1}(Q) = C'_{i22}(q_i) & \forall i \in I
\]  

Alternative to single-gas agreements may consist in an agreement that encompasses all the gases involved in the polluting process. This kind of agreements is referred hereafter as comprehensive agreements. Indeed, one can imagine two different settings in this case. The environmental agreement can specify targets (i) in aggregate level of abatement or (ii) in each gas separately. In the Kyoto Protocol, the first approach
is retained. However, we show in appendix that these two approaches are equivalent in our framework. Therefore, in the remainder of the paper, we use the latter definition, which allows more straightforward comparison with other kinds of agreements.

**DEFINITION 2** (Comprehensive agreement). A comprehensive agreement is given by the partition of \( \mathcal{P}(I) = \{ S, \{ i \}_{i \notin S} \} \) such that countries belonging to \( S \) choose cooperatively their abatement in gas \( g_1 \) and in gas \( g_2 \), while other countries behave like singletons. These decisions are assumed to take place simultaneously.

The corresponding problem is:

\[
P_{C_{(1,2)}} \left\{ \begin{array}{l}
\max_{(q_1, q_2) \in S} \pi_i (q_i, q_{-i}) \\
\max_{q_1, q_2} \pi_i (q_i, q_{-i}) \quad \forall i \in I \setminus S
\end{array} \right.
\]

That leads to the following first-order conditions:

\[
\sum_{k \in S} B'_k(Q) = C'_{i1}(q_{i1}) \quad \forall i \in S
\]

(8)

\[
\beta \sum_{k \in S} B'_k(Q) = C'_{i2}(q_{i2}) \quad \forall i \in S
\]

(9)

\[
B'_i(Q) = C'_{i1}(q_{i1}) \quad \forall i \in I \setminus S
\]

(10)

\[
\beta B'_i(Q) = C'_{i2}(q_{i2}) \quad \forall i \in I \setminus S
\]

(11)

We now define a new kind of environmental agreement. The ‘gas-by-gas’ agreement allows the presence of two co-existing agreements, each of them dealing separately with one pollutant. It is formally defined as follows:

**DEFINITION 3** (Gas-by-gas agreement). A gas-by-gas agreement is given by a set \( S_1 \) of countries, which choose cooperatively their abatement in gas \( g_1 \) and a set \( S_2 \) of countries, which choose cooperatively their abatement in gas \( g_2 \). Countries which do not belong to \( S_j \) set non-cooperatively their abatement in gas \( g_j \). These decisions are assumed to occur simultaneously.

The problem to be solved is thus the following:

\[
P_{G} \left\{ \begin{array}{l}
\max_{(q_1, q_2) \in S_1} \sum_{k \in S_1} \pi_k (q_k, q_{-k}) \\
\max_{q_1} \pi_i (q_i, q_{-i}) \quad \forall i \in I \setminus S_1
\end{array} \right.
\]

\[
\max_{(q_1, q_2) \in S_2} \sum_{k \in S_2} \pi_k (q_k, q_{-k}) \\
\max_{q_2} \pi_i (q_i, q_{-i}) \quad \forall i \in I \setminus S_2
\]
which first-order conditions are:
\[
\sum_{k \in S_1} B'_k(Q) = C'_{i1}(q_{i1}) \quad \forall i \in S_1 \tag{12}
\]
\[
\beta \sum_{k \in S_2} B'_k(Q) = C'_{i2}(q_{i2}) \quad \forall i \in S_2 \tag{13}
\]
\[
B'_i(Q) = C'_{i1}(q_{i1}) \quad \forall i \in I \setminus S_1 \tag{14}
\]
\[
\beta B'_i(Q) = C'_{i2}(q_{i2}) \quad \forall i \in I \setminus S_2 \tag{15}
\]

Using the specifications defined in equations (1) and (2), the first-order conditions of \( P^G \) become:
\[
\frac{1}{n} (a - Q) \gamma^{S_1,i} = q_{i1} \quad \forall i \in S_1 \tag{16}
\]
\[
\beta \frac{1}{n} (a - Q) \gamma^{S_2,i} = q_{i2} \quad \forall i \in S_2 \tag{17}
\]
\[
\frac{1}{n} (a - Q) \gamma^{i,i} = q_{i1} \quad \forall i \in I \setminus S_1 \tag{18}
\]
\[
\beta \frac{1}{n} (a - Q) \gamma^{i,i} = q_{i2} \quad \forall i \in I \setminus S_2 \tag{19}
\]

where:
\[
\gamma^{S,j}_{i,k} = \frac{B_S}{c_{ij}} = \frac{\sum_{k \in S} b_k}{c_{ij}} \tag{20}
\]

This problem is solved by summing aggregate emissions over the \( n \) countries \((q_{2i} \text{ is weighted by } \beta)\):
\[
Q^G = a \frac{A_1(S_1) + \beta^2 A_2(S_2)}{1 + A_1(S_1) + \beta^2 A_2(S_2)} \tag{21}
\]

where \( A_j(S_j) = \frac{1}{n} \left( \sum_{k \in S_j} \gamma^{S,j}_{i,k} + \sum_{k \in I \setminus S_j} \gamma^{k,j}_{i,k} \right) \)

Indeed gas-by-gas agreements lead to partition of \( I \) into four types of countries: those which belong to both agreements \((i \in S_1 \cap S_2, \text{ they are assigned a superscript } ^{ss})\), those which belong to only one agreement \((i \in S_1 \setminus \{S_1 \cap S_2\} \text{ and } i \in S_2 \setminus \{S_1 \cap S_2\}, \text{ denoted respectively } ^{sn} \text{ and } ^{ns})\) and those which do not belong to any agreement \((i \in I \setminus \{S_1 \cup S_2\}, \text{ denoted } ^{nn})\). Abatement levels depend on the partition \( \mathcal{P}(I) \). The aggregate abatement levels are given by equations (22) to (25).

\[
q^{ss}_i(\mathcal{P}(I)) = a \frac{\gamma^{S_1,i} + \beta^2 \gamma^{S_2,i}}{n + A_1(S_1) + A_2(S_2)} \tag{22}
\]
\[
q^{sn}_i(\mathcal{P}(I)) = a \frac{\gamma^{S_1,i} + \beta^2 \gamma^{i,i}}{n + A_1(S_1) + A_2(S_2)} \tag{23}
\]
\[
q_{i}^{ss}(\mathcal{P}(I)) = \frac{\alpha n + B_{S_{1}}} {n(1 + A_{1}(S_{1}) + A_{2}(S_{2}))} 
\]

\[
q_{i}^{sn}(\mathcal{P}(I)) = \frac{\alpha n + \beta \gamma_{i}^{2} S_{2,i}} {n(1 + A_{1}(S_{1}) + A_{2}(S_{2}))} 
\]

For such a given partition of \( I \), the net benefits are easily computable:

\[
\pi_{i}^{ss}(\mathcal{P}(I)) = a_{i}^{2} b_{i} \left( n - \frac{n + B_{S_{1}} S_{1,i} + \beta^{2} B_{S_{2}} S_{2,i}} {1 + A_{1}(S_{1}) + A_{2}(S_{2})} \right)^{2}
\]

\[
\pi_{i}^{sn}(\mathcal{P}(I)) = a_{i}^{2} b_{i} \left( n - \frac{n + \beta \gamma_{i}^{2} S_{2,i}} {1 + A_{1}(S_{1}) + A_{2}(S_{2})} \right)^{2}
\]

\[
\pi_{i}^{ns}(\mathcal{P}(I)) = a_{i}^{2} b_{i} \left( n - \frac{n + \gamma_{i}^{2} S_{2,i}} {1 + A_{1}(S_{1}) + A_{2}(S_{2})} \right)^{2}
\]

\[
\pi_{i}^{nn}(\mathcal{P}(I)) = a_{i}^{2} b_{i} \left( n - \frac{n + \gamma_{i}^{2} S_{2,i}} {1 + A_{1}(S_{1}) + A_{2}(S_{2})} \right)^{2}
\]

The leakage is embodied in the factors \( A_{j}(S_{j}) \) since a non-signatory’s payoff is affected by the partition of \( I \). The reaction functions of non-signatories and signatories are thus clearly non-orthogonal. The payoff of any country, which signs an agreement on gas \( g_{j} \), is increasing in the ratio of its own slope of marginal benefit \( (b_{i}) \) over the aggregate slope of marginal benefit within the coalition \( (B_{S_{j}}) \).

Outcomes of the other kinds of agreement are obtained by imposing appropriate restrictions on the partition of the set of countries. The outcome from a single-gas \( j \)-agreement, is thus computed by using the previous equations (26)-(29) and assuming that \( S_{-j} = \emptyset \).

Likewise, if the partition of \( I \) is assumed to be such that \( S_{1} = S_{2} = S \), one can find the outcome of the comprehensive \((1-2)\)-agreement.

1.3.2. First stage of the game: membership game

To find out sub-game perfect equilibrium of the two-stage game, we proceed by backward induction. Therefore, given the outcome of the emission game computed above, the equilibrium of the membership game consists in a partition of \( I \) such that unilateral deviation from a country is not individually profitable.

In the case of single-gas or comprehensive agreement, the choice that each country has to make in this first stage is binary, since the alternatives are lying between ‘signing’ or ‘not signing’ the treaty. Following Carraro and Siniscalco (1992), we use the internal and external
stability concepts first developed by d’Aspremont et al. (1983). These conditions require that an agreement is stable if: (i) a signatory country is better off when remaining inside the treaty rather than leaving it and (ii) a non-signatory country is better off when remaining outside of the agreement rather than in joining it. Moreover, a stable agreement is required to be profitable, that is to say that a signatory should be better off when signing the treaty than the non-cooperative outcome. To be stable, a partition of \( I \) \( (P^S(I) = \{ S, \{ i \}_{i \in I \backslash S} \}) \) is thus required to be such that:

\[
\begin{align*}
\pi^s_i(P^S) & \geq \pi^n_i(P^{S \backslash \{i\}}) \quad \forall i \in S \\
\pi^n_k(P^S) & \geq \pi^k_k(P^{S \cup \{k\}}) \quad \forall k \in I \backslash S \\
\pi^s_i(P^S) & \geq \pi^s_i(P^\emptyset) \quad \forall i \in S
\end{align*}
\]

where \( \pi^s_i(P^S) \) (resp. \( \pi^n_i(P^S) \)) is the net benefit of a signatory (resp. non-signatory) country \( i \) in the situation \( P^S \).

In the case of gas-by-gas agreements, the stability conditions are considerably enriched, since each country faces four possible moves.

2. Single-gas agreements in the homogenous case

We first examine single-gas \( j \)-agreements. As noted by Hoel in the above quoted assertion, this type of agreement is likely to be less efficient than comprehensive agreements. The issue being addressed in this section consists in assessing the outcome of an agreement that neglects one source of pollution.

For the sake of simplicity, we focus on the homogenous case. Then, the following assumptions are made:

\[
\begin{align*}
b_i &= b \quad \forall i \in I \\
c_{ij} &= c_j \quad \forall (i, j) \in I \times \{1, 2\} \\
\gamma^{i,j} &= \gamma_j \quad \forall (i, j) \in I \times \{1, 2\}
\end{align*}
\]

If \( |S| = s \), we then have:

\[
\gamma^{S,i}_j = s \gamma_j
\]

Moreover, the following normalizations are used:

\[
k = \frac{\beta^2 \gamma_2}{\gamma_1} = \frac{\beta^2 c_1}{c_2} \quad \text{and} \quad \gamma_1 = \gamma
\]

\( k \) thus stands for the relative slope of the marginal abatement costs related to a supplementary emission unit of \( g_1 \)-equivalent.
The assumption of symmetric countries allows for great simplification of this problem. The first one is that an agreement can be fully depicted by its size. Hence, the discussion of the stability of a given agreement is summed up to a discussion on the size of the agreement. Another important simplification allowed by the assumption of homogeneity is that any burden-sharing rule within the agreement leads to the same results.

2.1. Single gas agreement: outcome of the emission game

In order to obtain the results of the second-stage game, equations (22) and (25) are used and combined with the above assumptions. Single-gas 1-agreements are first considered. Hence, $S_2$ is assumed to be empty. Since the countries are assumed to be *ex ante* identical, the partition of the set $I$ is fully defined by the agreement size:

$$q^s_i(s_1, 0) = \frac{a\gamma(s_1 + k)}{n + \gamma(s_1(s_1 - 1) + n(k + 1))} \forall i \in S_1$$ (36)

$$q^n_i(s_1, 0) = \frac{a\gamma(k + 1)}{n + \gamma(s_1(s_1 - 1) + n(k + 1))} \forall i \in I \setminus S_1$$ (37)

Consistently to the intuition, signatories’ abatement is higher than non-signatories’ and the difference increases with respect to the number of signatories for low sizes of agreement. As mentioned above, the best-reply functions of the signatory and non-signatory countries are not orthogonal because of the non-linear setting of the benefit function. Hence, the abatement of a non-signatory country is decreasing with the size of the agreement.

The total abatement for a single-gas 1-agreement is thus:

$$Q(s_1, 0) = a\gamma \frac{s_1(s_1 - 1) + n(k + 1)}{n + \gamma(s_1(s_1 - 1) + n(k + 1))}$$ (38)

The signatory countries’ abatement is first increasing with respect to the size of the agreement. Over a given threshold value of $s_1$, the global reduction in the level of pollution is such that the abatement required from individual signatory is reduced by the entry of a new country, and, hence, $q^s_i(s_1, 0)$ is then decreasing with $s_1$.

For a given size of agreement, single-gas 1-agreement perform greater reduction in the total pollution than single-gas 2-agreement when the slope of marginal abatement cost for gas $g_1$ is lower than for gas $g_2$ expressed in the same unit (i.e. $k < 1$). The total abatement is the same in the two cases when $k = \frac{s_1(s_1-1)}{s_2(s_2-1)}$. The comparison between the abatement achieved through the single-gas agreements is illustrated in figure 1.
Figure 1. Abatement levels in the case of single-gas $j$-agreements (parameter values: $a = b = n = 100, c = 800, k = 0.5$).
The net individual benefits are:

\[ \pi_s^I(s_1, 0) = \frac{a^2 b}{2} \left( \frac{1}{n} - \frac{n + \gamma(s_1^2 + k)}{(n + \gamma(s_1(s_1 - 1) + n(k + 1)))^2} \right) \] (39)

\[ \pi_n^I(s_1, 0) = \frac{a^2 b}{2} \left( \frac{1}{n} - \frac{n + \gamma(1 + k)}{(n + \gamma(s_1(s_1 - 1) + n(k + 1)))^2} \right) \] (40)

The net benefit of a non-signatory country is clearly increasing in the size of the agreement as the environmental quality increases with \( s_1 \). This effect also holds for a signatory country, but it is partially offset by the increase in its abatement costs. Non-signatories are thus always better off than signatories. The net benefit are sketched for the two kinds of agreements in figure 2.

2.2. SINGLE GAS AGREEMENT: OUTCOME OF THE MEMBERSHIP GAME

Again, we first focus on single-gas 1-agreements and extend the results to single-gas 2-agreements. To address the stability issue, the following function, \( L(\cdot, 0) \), is used:

\[ L(s_1, 0) = \pi_s^I(s_1, 0) - \pi_n^I(s_1 - 1, 0) \] (41)

When \( L(s_1, 0) \) is negative, at least one signatory country would be better off in exiting the treaty in the situation \((s_1, 0)\). When it is positive, one non-signatory country would be better off in joining the environmental coalition in the situation \((s_1 - 1, 0)\). Hence, the \((s_1, 0)\)-agreement is stable when its size \( s_1 \) is such that \( L(s_1, 0) \geq 0 \) and \( L(s_1 + 1, 0) \leq 0 \).

PROPOSITION 1. \textit{Whatever the values of the parameters } \( a, b, \gamma, k \) \textit{and } \( n \geq 3 \), \textit{the size of any stable single-gas agreement cannot be greater than two.}

\textit{Proof.} see appendix ■

Although a partial environmental coalition can emerge as an equilibrium of the membership game, incentives to defect are too large when the size of the agreement is greater than three.\textsuperscript{13}

We now characterize the conditions that allow an agreement of size two to be stable. As the stability function is negative for \( s_j \geq 3 \), the stable agreement, if it exists, should correspond to a situation for which \( L(2, 0) \geq 0 \). Let denote the ratio of marginal abatement cost in each gas relatively to total marginal abatement cost by \( \delta_j \) (\( \delta_1 = \frac{1}{k+1} \) and \( \delta_2 = \frac{k}{k+1} \)).
Figure 2. Net benefits in the case of single-gas $j$-agreements (parameter values: $a = b = n = 100, c = 800, k = 0.5$).
PROPOSITION 2. A single-gas $j$-agreement of size two is stable if and only if $\gamma$ is sufficiently small ($k$ and $n \geq 3$ given), that is to say $\gamma \leq \left(\frac{n}{k+1}\right) / \left(\frac{n - 2(\delta_j + 1) + 2\sqrt{(n - \delta_j)^2 - 3\delta_j(n - 1)}}{n - 2(\delta_j + 1) + 2\sqrt{(n - \delta_j)^2 - 3\delta_j(n - 1)}}\right)$. If this condition is not fulfilled, the non-cooperative outcome prevails.

Proof. see appendix ■

COROLLARY 1. If $\gamma(k + 1) \leq \frac{1}{3}$, a stable single-gas $j$-agreement of size two exists.

Proof. It is sufficient to see that $\lim_{n \to +\infty} \gamma_j^+ = \frac{1}{3(k + 1)}$ and that $\gamma_j^+ \geq \frac{1}{3(k + 1)}$. ■

Notice that the threshold value of $\gamma_j^+$ depends on the weight of each gas in the marginal abatement costs expressed in $g_1$-equivalent ($\delta_1$ and $\delta_2$). Depending on the slope of marginal abatement cost in gas $g_j$ relatively to $g_{-j}$, stable single-gas $j$-agreement may exist while stable single-gas ($-j$)-agreement may not. However, since the difference between $\gamma_1^+$ and $\gamma_2^+$ decreases as $n$ increases (the other parameters remaining constant), the sets of parameter values for which single-gas agreements exist tends to be the same when $n$ is large.

The main result is that the maximum size of any stable single-gas agreement is two, and such an agreement holds when $\gamma$ is small. This result may appear to be rather counter-intuitive: whatever gas at stake in the negotiation process, a stable agreement may emerge if and only if the slope of marginal environmental benefit is not too large relatively to the slope of marginal abatement cost. Indeed, when $\gamma$ is large, the abatement level within the agreement is also large and so are the abatement costs faced by the signatories. Hence, the decrease in the pollution is not sufficient to offset the incentive to remain outside of the agreement.

3. Comprehensive agreements in the homogenous case

Let now consider the case of comprehensive agreements. This kind of agreement encompasses all the pollutants as the Kyoto Protocol does. That means that if a country decides to sign the agreement, he is required to reduce its emissions in all the gases. Reversely, if a signatory country decides to leave the agreement, no abatement in any gas is required. Again, we compute and analyze the outcome of the emission game and then examine the stability of such kind of agreements.
3.1. Comprehensive agreements: outcome of the emission game

The outcome of the comprehensive agreement is given by the equations (22)-(25) and (26)-(29) assuming that $S_1 = S_2 = S$ and that $|S| = s$.

The individual abatements are:

$$q_i^s(s, s) = \frac{a\gamma s(k + 1)}{n + \gamma(s - 1)(s + n)(k + 1)}$$  (42)

$$q_i^n(s, s) = \frac{a\gamma(k + 1)}{n + \gamma(s - 1)(s + n)(k + 1)}$$  (43)

The general interpretation of these results is not modified as compared to the comments made in the former section. Abatement levels from the signatory countries are $s$ time bigger than from the non-signatory countries, with $s$ given. As the size of the agreement increases, the non-signatory countries’ abatement decreases. For low sizes of agreement, the signatory countries’ abatement is increasing with respect to $s$ until $s \leq \sqrt{1 + \frac{n}{\gamma(k+1)}}$. Over this threshold, it is decreasing with respect to $s$. However, the total abatement remains strictly increasing with $s$, as the negative effect induced by the decrease in signatory countries’ abatement is offset by the entry of a new member. These features are sketched on figure 3 (left).

The individual benefits are:

$$\pi_i^s(s, s) = \frac{a^2b}{2} \left( \frac{1}{n} - \frac{n + \gamma s^2(k + 1)}{(n + \gamma(s - 1) + n)(k + 1)^2} \right)$$  (44)

$$\pi_i^n(s, s) = \frac{a^2b}{2} \left( \frac{1}{n} - \frac{n + \gamma(k + 1)}{(n + \gamma(s - 1) + n)(k + 1)^2} \right)$$  (45)

Figure 3 (right) shows the evolution of each country’s payoff with respect to the size of the comprehensive agreement. Two effects make the non-signatories better off when $s$ increases: (i) they benefit from the improvement in the environmental quality and (ii) at the same time, they face lower abatement cost. On the contrary signatories’ abatement cost increases at least for low values of $s$ and, hence, whatever $s$, non-signatories are better off than signatories. In the simulation showed in figure 3, signatories’ benefit is increasing, even for low values of $s$. This is ensured by the fact that $\gamma$ is not too large. This point is important in the subsequent discussion on stability.
Figure 3. Abatement levels (left) and net benefits (right) in the case of comprehensive agreements (parameter values: $a = b = n = 100$, $c = 800$, $k = 0.5$).
3.2. Comprehensive agreements: outcome of the membership game

The stability function in this case is defined as follows:

\[ L(s, s) = \pi^s(s, s) - \pi^n(s - 1, s - 1) \]  \hspace{1cm} (46)

The interpretation of the stability function is the same as in the former section. The difference lies in the commitment of any signatory country to achieve abatement in both pollutants.

**Proposition 3.** Whatever the values of the parameters \( a, b, \gamma, k \) and \( n \geq 3 \), the size of any stable comprehensive agreement cannot be greater than two.

*Proof.* see appendix ■

**Proposition 4.** A comprehensive agreement of size two is stable if and only if \( \gamma \) is sufficiently small, i.e.

\[ \gamma \leq \left( \frac{n}{k+1} \right) / \left( n - 4 + 2\sqrt{n^2 - 3(n-1)} \right) \]

\( (k \text{ and } n \geq 3 \text{ given}) \). If this condition is not fulfilled, the non-cooperative outcome prevails.

*Proof.* see appendix ■

**Corollary 2.** If \( \gamma(k + 1) \leq \frac{1}{3} \), a stable comprehensive agreement of size two exists.

*Proof.* It is sufficient to see that \( \lim_{n \to +\infty} \bar{\gamma}^+ = \frac{1}{3(k + 1)} \) and that

\[ \bar{\gamma}^+ \geq \frac{1}{3(k + 1)} \] ■

The general results regarding stability found in the former section also hold for comprehensive agreements. The stability of any comprehensive agreement involves low number of participating countries and is subject to the condition that \( \gamma \) is not too large. If not, the incentive to defect prevents any agreement of this kind from emerging as remaining outside of the agreement is an equilibrium strategy for all the countries.

3.3. Comparison between single-gas and comprehensive agreements

The comparison between the results above and those obtained in section 2 highlights the global and individual gains involved by the enlargement of the set of pollutants included in the treaty. It is obvious that a
(s, s)-comprehensive agreement allows to achieve greater $g_1$-equivalent abatement than any $(s, 0)$ or $(0, s)$ single-gas j-agreement.

As $k$ tends to zero (resp. $+\infty$), the difference between single-gas 1- (resp. 2-) agreements and comprehensive agreements decreases. Consequently, to assess the improvement in global welfare due to the enlarging of the set of pollutants encompassed in the treaty, it is necessary to consider the relative abatement costs for each gas expressed in $g_1$-equivalent ($k$), and not only the relative environmental impact of each gas ($\beta$).

From the point of view of the non-signatories, enlarging the treaty is unambiguously profitable given that they remain outside of the agreement and that the number of signatories remains constant. If so, the quality of the environment is improved while in the same time their abatement costs are lowered. However, as far as the signatories are concerned, the result is not as straightforward. Indeed, if extended in this way, the treaty would involve higher abatement for them and thus reduce their net benefit and increase the incentive to defect.

Nevertheless, corollary 3 shows that, in this case, comprehensive agreements better resist to free-riding incentives, since the conditions that define a stable agreement are less restrictive (see also figure 4).

**COROLLARY 3.** If two countries reach a single-gas j-agreement, then they can reach a comprehensive agreement. The reciprocal does not hold.

*Proof.* It is sufficient to notice that $\bar{\gamma}_j^+ \leq \bar{\gamma}^+$ for $j \in J$. Combined with propositions 2 et 4, these relations prove the corollary. ■

### 4. Gas-by-gas agreements

This section is devoted to the comparison of the results of gas-by-gas agreements with other kinds of agreements. The space of strategy available for each country is extended to account for the possibility of cooperating on one gas and not on the other.

#### 4.1. Graphical analysis of gas-by-gas agreements

Since four strategies are available to each country, any gas-by-gas is defined by the partition of $I$ into $S_1 \cap S_2$, $S_1 \{S_1 \cap S_2\}$, $S_2 \{S_1 \cap S_2\}$ and $I \{S_1 \cup S_2\}$. In the homogenous case that we analyse, a gas-by-gas agreement may be defined by the number $s_1$ of countries which cooperate on $g_1$, the number $s_2$ of countries which cooperate on $g_2$ and by $e$ as the number of countries which cooperate on both gases. Note however that the individual and global results of the emission game
Figure 4. Comparison of $\gamma$ values for which stable agreements exist within a gas-by-gas regime do not depend on the cardinal of $S_1 \cap S_2$, $e$ (see the appendix). A graphical representation of the set of gas-by-gas agreements in $\{1, n\}^2$ is given in figure 5.

PROPOSITION 5. The total abatement $Q(s_1, s_2)$ is increasing with respect to $s_1$ (resp. $s_2$) with $s_2$ (resp. $s_1$) given. The iso-abatement curves for $(s_1, s_2) \in I^2$ are strictly decreasing at a decreasing rate. The slope of iso-abatement curves are independent of $a$, $b$, $c$ and $n$ are increasing in $k$.

Proof. see appendix ■

This is illustrated in figure 5. The lower is $k$, the steeper are the iso-abatement curves. This rather intuitive result is due to the fact that a lower slope of marginal abatement cost in gas 1 comparatively to $g_2$ (in $g_1$-equivalent) required less countries to cooperate on $g_1$ to reach the same environmental result. Corollary 4 is directly derived from these properties.

COROLLARY 4. Consider any $(s_1, s_2)$ gas-by-gas agreement. It exists $s^* \in [s_1, s_2]$ such that a $(s^*, s^*)$ agreement achieves the same level of global abatement. The closer to $0$ (resp. to $+\infty$) is $k$, the closer is $s^*$ to $s_1$ (resp. to $s_2$).
Proposition 6. For the same environmental result, the global benefit that is reached under the \((s^*, s^*)\) agreement is higher than under any environmentally equivalent \((s_1, s_2)\) gas-by-gas agreement.
This is illustrated in figure 6 (right). Along each curve, the global benefit is constant. The difference between global benefit along two consecutive curve is also constant. It is easy to show that the slope of any iso-benefit curve is $-\frac{1}{k}$ when it crosses the 45 degrees-line. On the example shown, the gradient of the global benefit is high when $s_1$ and $s_2$ are low and is decreasing as $s_j$ tends towards $n$. The maximum benefit yield through cooperation on $g_2$ (for $s_2 = 100$) requires about fifty countries to cooperate on $g_1$ and only forty if the treaty encompasses the two gases.

Because of their cost-effectiveness perspective, comprehensive agreements do better than any other gas-by-gas agreements. However, the other side of the argument presented in proposition 6 is also noteworthy. Consider a comprehensive $(s, s)$ agreement and operate a move along a iso-profit curve towards any $(s_1, s_2)$ agreement. The net global benefit is obviously the same, whereas the environmental quality is improved. Of course, this move requires the entry of non-signatories into $S_1$ and/or $S_2$.

4.2. **Membership game: an example**

The results found in sections 2 and 3 suggest that stability requirement strongly restricts the feasible set of international agreements to a few participating countries. In the case of gas-by-gas agreements, stability
conditions are complicated by the fact that four types of strategies are available to each country. Thus, even in the homogenous case, since each country faces three alternative choices, twelve conditions are required to be fulfilled simultaneously for general partition of $I$ (less if some strategies are not used by the countries).

To illustrate the possible second-stage outcomes, we discuss a numerical example. The parameters are such that the conditions exposed in corollaries 1 and 2 are fulfilled. Hence, the single-gas agreements defined by $(s_1, s_2) = (2, 0)$ and $(s_1, s_2) = (0, 2)$ are stable. The first yields higher global benefit and higher global abatement than the second since $k < 1$. The comprehensive $(2, 2)$ agreement is also stable as it is not profitable for a signatory to defect $(1450, 2 > 1447, 6)$ while it is also not profitable for a non-signatory to join the agreement $(1470, 0 > 1461, 8)$. On this example, we also see that, starting from any single-gas $j$-agreement, both signatories and non-signatories would benefit from extending the treaty to gas $g_{-j}$ and negotiating a comprehensive agreement.

However, introducing gas-by agreements widens the set of possible outcomes. If a single-gas 1-agreement is signed, it is in non-signatories’ interest to propose another agreement that deals uniquely with gas $g_2$ $(1463.4 > 1462.6)$. In this case, the situation where two countries form a coalition on gas $g_1$ and two other countries cooperate on $g_2$ is stable, since no country faces incentives to change its decision in the membership game. Although this situation leads to the same global results than a comprehensive agreement, no country would accept in this case to sign a comprehensive agreement.

### Concluding remarks

In the case of climate change, the difficulty to achieve full cooperation is strengthened by the number of pollutants involved in the polluting process. Our results partly show how the outcomes of a climate negotiation could be modified when the “basket of pollutants” is extended. This plays an important role in the economic and environmental results of the treaty, but also in the incentives to join the treaty or not.

From a cost-effectiveness perspective, it is unambiguously profitable to enlarge the set of pollutants included in an international agreement. Our results show that the superiority of comprehensive agreements also lies in the fact that such agreements better resist to incentive to defect than any single-gas agreement. Indeed, we have shown that for some set of parameters, self-enforcing comprehensive agreements may emerge as an equilibrium, while single-gas agreements may not.
Table I. Example of a gas-by-gas agreement payoffs (parameter values: $a = b = n = 100, c = 800, k = 0.5$).

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<tr>
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As soon as the decision to enlarge the "basket of pollutants" is made, arises the question of the equivalence rule between the pollutants. So far, the Global Warming Potential (GWP) index is used to convert emissions of different gases into CO$_2$ equivalent. Since this index relies only on scientific foundations, it does not give a proper account of the relative values of the different pollutants. From an economic perspective, this index is questionable for at least two reasons: (i) it does not account for the differences between uncertainties concerning the differences in the long-run impact of abatement in each gas $^{14}$ (ii) since no discount rate is used, the question of the inter-temporal choice of
abatement in the different gases is neglected and this rule may mislead economic choices.\textsuperscript{15} Our results stress the importance of this parameter on the set of stable agreements.

'Gas-by-gas' agreements – that is to say the possibility for agreements on each pollutant to coexist – could be a way to induce new countries to join the environmental agreement insofar as it could provide them with sufficient incentives. This result may be strengthened by relaxing the assumption of homogeneity. Indeed, considering heterogenous countries regarding abatement cost in the different gases may ease the emergence of dissymetric gas-by-gas agreements. Further work is needed in this direction.

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Notes

\textsuperscript{1} Numerous studies have extended these results, examining the influence of heterogeneity and burden-sharing rules among the signatories (Barrett, 1997a; Botteon and Carraro, 1997), different equilibrium concepts (Barrett, 1994; Ecchia and Mariotti, 1998; Carraro and Moriconi, 1998), and different specifications of the payoff functions (Carraro and Siniscalco, 1993; Heal, 1994; Péreau and Tazdait, 2000; De Cara and Jayet, 2001). An alternative approach, based on cooperative-game concepts has been proposed by Chander and Tulkens (1992, 1997). These authors show that a supra-national agency can sustain Pareto-optimality through an appropriate transfer scheme among all countries. This transfer scheme presents the interesting property of lying into the core and thus resists to coalitional free-riding. However, this approach requires the existence of a supra-national body, which does not hold in reality (for a comparison of the two approaches see Tulkens (1997)

\textsuperscript{2} As soon as 1995, methane and nitrous oxide emissions are included in the targets first sketched in Rio. In Kyoto, the set of pollutants has been extended to include three other trace-gases (HFC, PFC and SF\textsubscript{6}).

\textsuperscript{3} The controversy on accounting or not carbon storage in net emissions relies partly on the fact that the carbon sequestered may be released in the future, whereas reduction in emissions have a permanent effect (Feng et al., 2000).

\textsuperscript{4} The global warming potential index is computed and published by IPCC for different time horizons (20, 100 and 500 years) (Intergovernmental Panel on Climate
Multi-Greenhouse Gas International Agreements

Change, 1995). Indeed, the 100-year GWP is often retained as the reference in international negotiations. However, this index varies in a quite wide range for different time horizons. The main reason for that is that the lifetime in the atmosphere differs widely from one gas to another.

Consider for instance that \( g_1 \) stands for CO\(_2\) and \( g_2 \) for CH\(_4\). \( \beta \) is then simply the GWP of CH\(_4\) (\( \beta = 21 \) in this case).

Carraro (1998) reviews different membership rules for the first stage of the game. Chander and Tulkens’s cooperative approach can be viewed as a unanimity membership rule. The assumption that the membership decisions are simultaneous is also different from the sequential game proposed by Bloch (1997).

Note, however, that in the original version of this model by Barrett (1994), the leakage plays an ambiguous role. Indeed, Barrett assumes a second-stage Stackelberg game, in which the signatory countries act jointly as the leader. This tends to increase the benefit of the signatories because they expect greater abatement from the non-signatories. Hence, without non-orthogonal best-reply functions, large stable agreements as found by Barrett are not possible (see also Barrett (1997b)).

Note that, even if not included in the treaty, each country takes into account the influence of gas \( g_{-j} \) in the computation of its own net payoff.

This concept is formally defined in Chander and Tulkens (1997).

Without loss of generality, these conditions are written for a single-gas 1-agreement. The results in the case of a single-gas 2-agreement are straightforward by using appropriate change in the meaning of \( \beta \).

Indeed, due to the reference to Nash equilibrium, it is equivalent to consider that \( S_j \) is empty or reduced to a singleton.

In the case of heterogeneous countries, the burden-sharing rule within the agreement is not straightforward as soon as side payments between the signatories are allowed. Barrett (1997a) and Botteon and Carraro (1997) have examined this problem and compared different type of bargaining rule (Nash bargaining, Shapley value). The results are quite sensitive to the choice of this rule.

Barrett (1994) finds larger sizes of stable environmental coalition, which can be as large as the total number of countries. But once again, this result is due to the Stackelberg assumption that provides greater incentives to reach the agreement. However, even in this case, the environmental coalition is large only when the difference between the non-cooperative and full-cooperative global benefit are very small. As soon as the decisions in the first-stage game are simultaneous, the low size of the environmental coalition is robust to different specifications of the net benefit function (De Cara and Jayet, 2001).

This reason is often used as an argument not to include carbon sinks in the Kyoto targets. As a matter of fact, the environmental impact of storing carbon depends on the future use of wood and, to this respect, is not strictly equivalent to reduce emissions by the same amount (Feng et al., 2000).

One may admit, that, in this case, the difficulty then lies in the choice of the appropriate discount rate.
References


Appendix

*Alternative comprehensive agreements.* An alternative type of comprehensive agreement is considered. Hence:

**Definition 4 (Comprehensive (1-equivalent)- agreement).** A comprehensive (1-equivalent)-agreement is given by the partition of $P(I) = \{S, \{i\}_{i \in I \setminus S}\}$ such that countries belonging to $S$ choose cooperatively their aggregate abatement $\{\tilde{q}_i\}_{i \in S}$ expressed in terms of $g_1$-equivalent, while other countries behave like singletons. These decisions are assumed to occur simultaneously.

Provided that each signatory country minimizes his total abatement cost subject to the aggregate abatement constraint, the problem faced
by a signatory country $i$ is as follows:

$$\min_{q_1, q_2} C_{i1}(q_1) + C_{i2}(q_2)$$

subject to $q_1 + \beta q_2 \geq \bar{q}_i$.

Country $i$ would thus tend to equate the marginal abatement costs in each gas (expressed in the same unit) and would choose $q_1$ and $q_2$ in order to make the abatement constraint binding. That is to say: $\beta C'_{i1} = C'_{i2}$ and $q_1 + \beta q_2 = \bar{q}_i$. Thus, the abatement mix for a given target of abatement $\bar{q}_i$ is the following:

$$q_1 = \frac{c_{i1}}{\beta c_{i1} + c_{i2}} \frac{\bar{q}_i}{c_{i2}} \quad (49)$$

$$q_2 = \frac{\beta c_{i1}}{\beta c_{i1} + c_{i2}} \frac{\bar{q}_i}{c_{i2}} \quad (50)$$

The emission game is thus described by the following problem:

$$\max_{(q_i)_{i \in S}} \left\{ \sum_{k \in S} \pi_k(q_k, q_{-k}) \right\}$$

subject to (49) and (50)

that leads to first-order conditions that are equivalent to those for $P^{C(1-\gamma)}$.

**Proof of proposition 1.** Let first consider single-gas 1-agreements. Note that any coalition of size 3 leads to a negative stability function:

$$L(3,0) = -\frac{4a^2b(n(\gamma(n+3)+(n-1))+k(2\gamma(n(n+1)-2)+n(n-1))+k^2(\gamma n(n-1)))}{(n+\gamma(2+n(k+1)))^2(n+\gamma(6+n(k+1)))^2} \leq 0$$

It is sufficient to see that the stability function is negative for $s_1 \geq 3$. Since non-signatory payoff increases more rapidly with $s_1$ than signatory payoff, the stability function is negative for all $s_1 \geq 3$. Hence, each signatory country faces a positive incentive to defect when $s_1 \geq 3$. The same holds for a single-gas 2-agreement with appropriate change in the meaning of $k$. ■

**Proof of proposition 2.** We look for conditions on the parameters allowing the stability function $L(2,0)$ to be positive. We thus solve $L(2,0) = 0$ with respect to $\gamma$. $L(2,0)$ can be rewritten as follows:

$$L(2,0) = \frac{a^2b(k+1)((k+1)n(3n-4)-4)}{2n^2(1+\gamma(k+1))^2(n+\gamma(2+n(k+1)))^2} \gamma(\gamma_1^+ - \gamma)(\gamma - \gamma_1^-)$$

(52)
where:

\[ \gamma_1^+/= \left( \frac{n}{k+1} \right) / \left( n - 4 + 2 \left( \delta_2 \pm \sqrt{(n-\delta_2)^2 - 3\delta_1(n-1)} \right) \right) \]  

(53)

Only \( \gamma_1^+ \) is positive. Then, \( L(2, 0) \geq 0 \) if and only if \( 0 \leq \gamma \leq \bar{\gamma}_1^+ \). Note that since \( \pi^s(1, 0) = \pi^n(0, 0) \) –the non-cooperative payoff–, this condition also ensures that the profitability condition is fulfilled.

Following the same reasoning for single gas 2-agreements, we find that a necessary and sufficient condition for \( L(0, 2) \geq 0 \) is that \( 0 \leq \gamma \leq \bar{\gamma}_2^+ \) with \( \bar{\gamma}_2^+ \) defined as follows:

\[ \bar{\gamma}_2^+ = \left( \frac{n}{k+1} \right) / \left( n - 4 + 2 \left( \delta_1 \pm \sqrt{(n-\delta_1)^2 - 3\delta_2(n-1)} \right) \right) \]  

(54)

These thresholds define the value of \( \gamma, n \) and \( k \) for which a single-gas \( j \)-agreement of size two is stable.

\[ \blacksquare \]

Proof of proposition 3. The proof follows exactly the same argument as in proposition 1.

\[ L(3, 3) = -\frac{4a^2b\gamma^2(k+1)^2n(n-1+\gamma(k+1)(n+3))}{(n+\gamma(k+1)(2+n))^2(n+\gamma(k+1)(6+n))^2} \leq 0 \]  

(55)

The negativity of \( L(s, s) \) for \( s \geq 3 \) indicates that a signatory country faces a positive incentive to leave the agreement.

\[ \blacksquare \]

Proof of proposition 4. The reasoning\(^1\) is the same as in proposition 2. We rewrite \( L(2, 2) \) as follows:

\[ L(2, 2) = \frac{a^2b(1+k)^3(n-2)(3n+2)}{2n^2(1+\gamma(k+1))^2(n+\gamma(k+1)(n+2))^2}\gamma(\gamma^+ - \gamma)(\gamma - \gamma^-) \]  

(56)

where \( \bar{\gamma}^+/= \left( \frac{n}{k+1} \right) / \left( n - 4 \pm 2\sqrt{n^2 - 3(n-1)} \right) \). \( \bar{\gamma}^+ \) is the only strictly positive root.

\[ \blacksquare \]

Proof of proposition 5. The proposition results from the study of the following partial derivatives:

\[ \frac{\partial Q}{\partial s_1}(s_1, s_2) = \frac{a\gamma n(2s_1 - 1)}{(n + \gamma(s_1(s_1 - 1) + ks_2(s_2 - 1) + n(k + 1)))^2} \]

\[ \frac{\partial Q}{\partial s_2}(s_1, s_2) = \frac{a\gamma n(2s_2 - 1)}{(n + \gamma(s_1(s_1 - 1) + ks_2(s_2 - 1) + n(k + 1)))^2} \]

\(^1\) \( L(s, s) \) can be written as a polynomial expression of \( s \), for which \( s = 1 \) is an obvious solution of \( L(s, s) = 0 \) because \( \pi^s(1, 1) = \pi^n(0, 0) \).
The slope of an iso-abatement curve is given by:

\[
\frac{ds_2}{ds_1} = -\frac{\partial Q}{\partial s_1}(s_1, s_2) - \frac{1}{k} \frac{2s_1 - 1}{2s_2 - 1}
\]

(57)

It is negative, decreasing with \( s_1 \) and increasing with \( k \). On the 45 degrees-line the slope is equal to \(-\frac{1}{k}\). ■

**Proof of proposition 6.** \( s^* \) is defined as the solution of \( Q(s^*, s^*) = Q(s_1, s_2) \).

\[
\Pi(s^*, s^*) - \Pi(s_1, s_2) = \frac{2ka^2\gamma(s_1 - s_2)^2(s_1 + s_2 - 1)(Y - 1)(s_1 + s_2 - 1) + 2s_1s_2}{(k + 1)(2s_1 - 1 + Y)(2s_2 - 1 + Y)(\gamma((k + 1)n + (s_1 - s_2^2 + k(s_2 - s_2^2))^2 + n)} \geq 0
\]

This difference is obviously strictly positive for \( s_1, s_2 \in I \) and \( s_1 \neq s_2 \). ■

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