Global Warming Potential, an imperfect but second best metric for climate change.

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Abstract. The paper revisits the concept of Global Warming Potentials (GWP) from an economic perspective and investigates the implications of GWP-based economic instruments.. We develop a general dynamic model to analyze multi-greenhouse gas issues. The results confirm the GWP induces a bias. We underline the trade-off between the time-horizon used in GWP computation and the discount rate: the GWP implicitly defines different discount rates for different gases. We also discuss the evolution of the shadow price ratio and present the analytical general solution of the economic-ecological system. A general expression of the second-best tax based on a given is derived.

Keywords: Global Warming Potential; Climate Change; Second-best instruments.

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1. Introduction

The ratification of the Kyoto Protocol by Russia turned all the speculation into reality and gave the protocol the force it will need. This cleared the way for the adoption of the so-called 'Kyoto flexibility instruments'. On November 18, 2004, the UNFCCC[1] registered the first Clean Development Mechanism (CDM): *Brazil NovaGerar Landfill Gas to Energy* (EcoSecurities, Ltd, 2004).[2] Although carbon dioxide is the most scrutinized greenhouse gas (GHG) in the climate change debate, CO_2 emissions are only indirectly targeted in this very first project. More surprisingly, the project proposes to *emit* –rather than abate– CO_2. Indeed, the project entails collecting landfill methane to produce electricity. Emission reductions are granted on the basis of *(i)* methane conversion into carbon dioxide, and *(ii)* the replacing of fossil fuel use in electricity generation. The key point in this project lies in the fact that methane's Global Warming Potential (GWP) is 21 times higher than that of CO_2, while the combustion reaction transforms each ton of methane into only 2.75 tons of CO_2 (or 44/16, the molar mass ratio). Hence, just through the conversion from a higher- into a lower-GWP gas, total GHG emissions are reduced approximately eightfold (from 21 to 2.75) on a CO_2-*equivalent* basis. This example is far from being anecdotal; a number of projects currently under review by the UNFCCC reap advantage from the same kind of between-gases arbitrages.

The above example illustrates well the importance of non-CO_2 gases for the design of economic instruments. The multi-pollutant nature of climate change has prompted a fierce debate over the agreement architecture, especially with regard to the inclusion of non-CO_2 GHGs and the delineation of the 'Kyoto basket of gases' (Article 5.3 of the Kyoto Protocol). Empirical studies have clearly established that multi-gas mitigation strategies economically dominate CO_2-only strategies. The underlying intuition is easy to understand: multi-gas targets make it possible to take advantage of the most cost-effective abatement options, and thus lower the cost of achieving any given reduction. The magnitude of the estimated cost-savings, although varying with models and assumptions, unambiguously favors multi- over single-gas strategies (Hayhoe et al., 1999; Reilly et al., 1999; Burniaux, 2000; Jensen and Thelle, 2001).

[1] United Nation Framework Convention on the Climate Change
[2] The interested reader is referred to `http://cdm.unfccc.int/Projects` for further information about this and other CDM projects.

The equivalence rule used in those studies for GHG comparison purposes is based on the GWP index. From the first example cited, one easily foresees the key role that this index plays in setting 'relative prices' for greenhouse gases. Direct climate impact and atmospheric lifetime greatly vary from one gas to another. The GWP reflects the time-integrated radiative forcing resulting from one emission pulse of any GHG relative to that of CO_2 (Intergovernmental Panel on Climate Change, 2001). The Kyoto Protocol, which made its use mandatory in the reporting of States' emissions, gave the GWP a status with regard to international law that few other physics concepts can claim.

Is the GWP index a good indicator to compare greenhouse gases from an economic perspective? In short, the answer is no. The concept behind GWP raises a number of issues. A number of these issues are equally raised when dealing with indicators that attempt to aggregate "apples and oranges" (OECD, 2002, replace here apples by carbon dioxide, and oranges by methane). Some authors have pointed out the simplified representation of the climate system the GWP relies on, questioning the relevance of this metric as an accurate climate change indicator (Smith and Wigley, 2000; Fuglestvedt et al., 2003; Godal, 2003). But the most fundamental criticisms are based on economic arguments. A small but growing amount of research has pointed out fundamental shortcomings in the GWP definition whenever this concept is used in cost-effectiveness assessments of multi-gas mitigation strategies. The lack of discounting, the overlooking of non-linearities in damage functions, and arbitrarily choosing time-horizons are the most often raised criticisms (Reilly and Richards, 1993; Kandlikar, 1996; Manne and Richels, 2001; Moslener and Requate, 2001; Tol et al., 2003). Arguably, these shortcomings lead to distortions in the economic value of abatements in various greenhouse gases, in particular between short- and long-lived greenhouse gases. As a direct consequence, policies based on the GWP concept are accused of misleading the time path of resource allocation in abatement efforts.

Yet, since its introduction in the first IPCC assessment report in the early nineties (Lashof and Ahuja, 1990), the vast majority of assessments and emission reports that had to deal with multi-greenhouse gas issues have relied on the GWP. The GWP still stands as a key-concept in the toolbox of policy-makers and climate scientists alike. In fact, the GWP has proved both more effective and more operational as a negotiation basis than any alternative index found in the literature. One possible reason for this may be that, rightfully or not, it is easier to reach an agreement on an index summarizing the radiative forcings and atmospheric lifetimes, which are well-documented and scientifically-sound, "hard" facts, than on an index that heavily depends on an economic measure of climate-change related damages and long-term discount rates.

The question addressed in this paper is: To what extent can GWPs be used in the design of climate change economic instruments? The paper investigates the economic implications of the use of GWP-based economic instruments. Some recent contributions have estimated the costs associated with Global Warming Potentials (O'Neil, 2003; Aaheim et al., 2004; Kurosawa, 2004) from an empirical perspective. However, few papers in the literature have specifically addressed the design of economic instruments based on a biased index such as the GWP. In this text, we present a fairly general multi-gas dynamic framework to analyze the question. Unlike most of the GWP literature that relies on cost-minimizing models (Kandlikar, 1996; Moslener and Requate, 2001) and assume separable abatement costs, the model developed in this paper provides a general framework for analyzing substitutions in the non-environmental part of surplus.

The paper is organized as follows. Section 2 revisits the GWP concept from an economic perspective, and reviews the main arguments critical views of to the GWP. Section 3 presents the analytical framework. We develop a general, m goods, n gases model to analyze multi-greenhouse gas issues in a dynamic setting. In the particular case of a linear relationship between damage and concentrations, we underline the fundamental trade-off between the time-horizon chosen in the computation of GWP and the discount rate. This analysis shows that an un-discounted measure based on a finite time-horizon implicitly defines different discount rates for different gases. We also discuss the evolution of the shadow price ratio given a general case and present the analytical general solution of the economic-ecological system. We then turn to the design of second-best price instruments based on the GWP index. We derive a general expression of the second-best tax based on a given metric. In particular, we show that the second-best tax can be computed as linear combinations of marginal damage.

2. Global Warming Potentials: An economic perspective

The first example given in the introduction highlights the importance of the equivalence rule used for GHG comparison purposes. When multi-gas targets are under consideration, they have to be formulated in a common unit, e.g., in tons of CO_2-equivalent. A metric is thus needed to compare GHGs. Fuglestvedt et al. (2003) provide an excellent and comprehensive survey of existing and alternative metrics of climate change. Among these metrics, the GWP is the most commonly used. This index was originally derived from a methodology developed for comparing ozone-depleting substances (Lashof and Ahuja, 1990).

It is worth having a closer look at the way GWPs are computed. Following the Intergovernmental Panel on Climate Change (2001), the definition of the concept is as follows:

$$\mathrm{GWP}_{j,\mathrm{CO_2}}(\hat{T}) = \frac{\int_0^{\hat{T}} c_j(t)\omega_j(t)dt}{\int_0^{\hat{T}} c_{\mathrm{CO_2}}(t)\omega_{\mathrm{CO_2}}(t)dt} \tag{1}$$

where $c_j(t)$ and $c_{\mathrm{CO_2}}(t)$ represent the remaining atmospheric quantities of gas j and CO_2, respectively, at time t after an emission pulse of one mass unit at time $t = 0$. $\omega_j(t)$ and $\omega_{\mathrm{CO_2}}(t)$ represent the instantaneous radiative forcing.

At first glance, the GWP concept might be regarded as a matter of interest for climate scientists and not for economists. As a matter of fact, this metric plays a crucial economic role, which is well illustrated in the first example discussed in the introduction: it determines the relative 'prices' at which reductions in various GHGs can be traded.

In an insightful analysis of the debate over the relevance of the GWP index, O'Neil (2000) has identified two major difficulties. The first difficulty has to do with the selection of a criterion/variable in the long causality chain that proceeds from emissions to the economic costs of climate damages. This causality chain is represented in Figure 1, which builds on analyses by O'Neil and Fuglestvedt et al.. The GWP concept is centered around an early link in this chain (radiative forcing). In contrast, the index proposed by Kandlikar (1996), based on the ratio of the marginal social values of concentration in each GHG, is built up from the end of the causality chain (marginal economic damages and abatement costs). Alternative indices that have been proposed in the literature fall somewhere in between these two options (Manne and Richels, 2001; Bradford and Keller, 2000). It should therefore come as no surprise that indices based on measures of different variables lead to diverging results.

One conclusion drawn from this discussion could be that this is less the GWP concept itself, rather than its use in cost-effectiveness assessments, that is problematic. The GWP simply cannot accurately describe variables it was not meant to measure in the first place, unless very restrictive assumptions are made on the subsequent links in the causality chain. This is the main argument used by Kandlikar, who highlights the implicit economic assumptions needed to have equivalence between the GWP and the optimal ratio of the shadow prices. Unsurprisingly, these assumptions are a zero discount rate as well as linearity in the relationship between temperature change and economic damages. Kandlikar argues that these assumptions are hardly justified in the case of climate change. Again, this is just the reflect of the discrepancy between the very meaning of GWP, i.e., equivalence at one particular link, and its use in cost-effectiveness analyses.

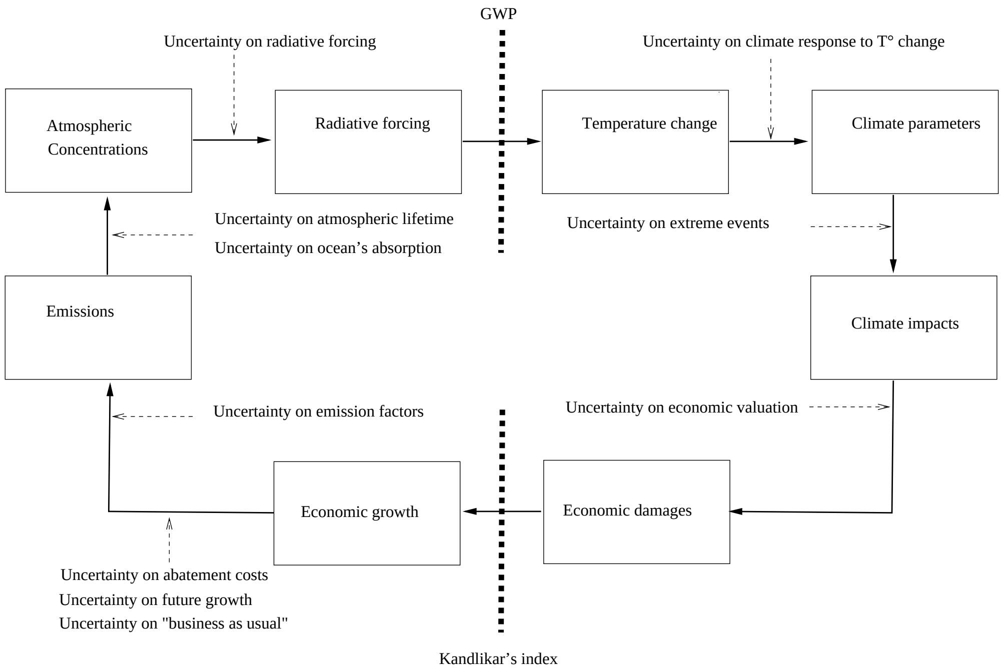


Figure 1. Causality chain

The second difficulty lies in how the variable of interest should be measured. Are instantaneous measures appropriate? Should time-integrated measures be preferred? If the latter is chosen, what is the appropriate time horizon? Acknowledging the essentially dynamic nature of climate change, most indices, such as Kandlikar's index and the GWP, rely on time-integrated measures. But, in contrast to the GWP index, Kandlikar uses a discounted measure. Arguably, relying on an un-discounted and constant metric leads to an error in the time-path allocation of abatements between short- and long-lived GHGs (see also Tol et al., 2003 and Michaelowa, 2003). The same argument is put forward by Moslener and Requate (2001), who find non-monotonic optimal trajectories of relative prices, emissions, and stocks. Consequently, un-discounted, constant metrics, are shown to induce sub-optimal abatement paths.

A third major difficulty has been widely overlooked in the above-mentioned literature. Uncertainties are compounded at all stages of the causality chain (see figure 1). From a practical perspective, the computation of an index that accurately reflects the economic conditions of optimality, whereby marginal abatement costs should be equal to marginal damages at all times, is complex. In addition to a full knowledge of the climate parameters, the computation of Kandlikar's index requires an economic valuation of the flow of marginal damages caused by climate change. Yet, huge uncertainties still surround the flow of climate impacts to be expected, say, over the next century, let alone the controversies about their translation into economic terms. Therefore, in order to be operational, indices such as Kandlikar's have to rely on an approximation of the functional form relating economic damages with average temperature changes. It still remains to be seen if this approximation induces smaller distortions than the bias caused by the GWP.

Moreover, all climate models agree on the fact that neither climate change impacts nor economic losses will be uniformly distributed across regions. Pushing Kandlikar's reasoning to its limit, the index should also reflect differentiated damages across regions. Recent developments in climate negotiations have shown how difficult it could be to reach a global agreement on global emission reduction targets. Although economically sound, "optimal" multi-gas indices might thus require too much information to serve as an operational negotiation basis.

3. Analytical framework: A multi-gas, optimal control problem

3.1. Notations

Vectors and matrices are in bold, lowercase and uppercase respectively. All vectors are column vectors unless otherwise stated. $\mathbf{A}^T$ denotes the transposed matrix of $\mathbf{A}$. The gradient of a mapping function g from $I\!R^n$ to $I\!R$ evaluated in $\mathbf{x}_0$ is the n-vector of the partial derivatives of g with respect to x_i and is denoted by $\boldsymbol{\nabla}_{[g,\mathbf{x}]}(\mathbf{x}_0) = \left(\frac{\partial g}{\partial x_i}(\mathbf{x}_0)\right)_{i=1,\ldots,n}$. The Hessian matrix of g evaluated in $\mathbf{x}_0$ is the $n \times n$-matrix of second-order partial derivatives: $\mathbf{H}_{[g,\mathbf{x}]}(\mathbf{x}_0) = \left(\frac{\partial^2 g}{\partial x_i \partial x_k}(\mathbf{x}_0)\right)_{i,k=1,\ldots,n}$. The Jacobian matrix of a mapping function $\mathbf{f}$ from $I\!R^n$ to $I\!R^m$ evaluated in $\mathbf{x}_0$ is the $m \times n$-matrix denoted by $\mathbf{J}_{[\mathbf{f},\mathbf{x}]}(\mathbf{x}_0) = \left(\frac{\partial f_i}{\partial x_k}(\mathbf{x}_0)\right)_{i=1,\ldots,m;k=1,\ldots,n}$.

3.2. Formulation of the general problem

Consider an economy with a set of (private) goods ($i = 1, \ldots, m$). Equilibrium quantities at time t is denoted by the m-vector $\mathbf{x}_t = (x_{1t}, \ldots, x_{mt})^T$. Consumption and/or production causes emissions of a set of gases ($j = 1, \ldots, n$) in quantities $\boldsymbol{\varepsilon}(\mathbf{x}_t) = (\varepsilon_1(\mathbf{x}_t), \ldots, \varepsilon_n(\mathbf{x}_t))^T$. Although some of the results in this paper are established with a general formulation of the relationships between the emissions and $\mathbf{x}$, the linear case whereby $\boldsymbol{\varepsilon}(\mathbf{x}) = \mathbf{E} \cdot \mathbf{x}$ will be given particular attention. In the case of a linear relationship between emissions and

$\mathbf{x}_t$, $\mathbf{E} = (e_{j,i})_{j=1,\dots,n;i=1,\dots,m}$ is the $n \times m$-matrix of the (constant) emissions factors that transforms good i into emissions in gas j.

Greenhouse gases are stock pollutants. We consider here the change in atmospheric concentrations, i.e., between current and pre-industrial levels. For gas j at date t, this quantity is denoted by z_{jt}, with $\mathbf{z}_t = (z_{1t}, \dots, z_{nt})^T$. The equation of motion of z_j is given by the following equation:

$$\dot{z}_{jt} = -\tau_{jt} z_{jt} + \varepsilon_j(\mathbf{x}_t) \quad (j = 1, \dots, n) \tag{2}$$

The two terms in equation (2) reflect the natural and anthropogenic components, respectively, in the accumulation of gas j in the atmosphere. Through natural absorption processes, z_{jt} decreases over time at rate τ_{jt}. In other words, if anthropogenic emissions are zero, atmospheric concentrations return to pre-industrial equilibrium levels at rate τ_{jt}. In general, τ_{jt} is not constant over time as it results from complex interactions between chemical species in the atmosphere, and might depend on the composition of the atmosphere itself. However, little is known about the functional form of τ_{jt}. In the rest of this paper, τ_j will be assumed constant. Average lifetime of gas j in the atmosphere is thus $1/\tau_j$.

The non-environmental part of the economic surplus is measured through $S(\mathbf{x}_t)$. $S(.)$ is assumed to be continuously differentiable. As this paper is focused on environmental aspects, the analytical form of the non-environmental part of the surplus is not detailed here. It is however assumed that there exists a unique vector $\mathbf{x}_t^{\mathrm{LF}}$ (LF as in *laissez-faire*) that maximizes $S(.)$. For all values of $\mathbf{x}$ that will be examined thereafter, we assume that $S(.)$ is increasing[3] with respect to all x_i ($S'_{x_i} > 0$).

All n gases are assumed to impact climate. This impact is summarized through the change in average global temperature, denoted by θ. Generally speaking, the impact on average temperature depends on the concentrations of all gases and is denoted by $\theta(\mathbf{z})$. Each gas j contributes to the increase in temperature through its radiative forcing $\theta'_{z_j}(\mathbf{z})$, which is assumed to be positive.[4]

Global economic damages are denoted by $D(\theta)$. Following a commonly used assumption in the literature, $D(.)$ is assumed to be increasing and convex with respect to θ ($D'(.) >$

[3] Indeed, in a general approach, it would be possible to also deal with positive externalities, not only with negative ones. This would be the case, for instance, for carbon sequestration, which offsets some of the emissions and thus contributes to lower the pressure on climate. In this case, sequestered carbon can be accounted for as new 'gas' (negative emissions). The reasoning would remain unchanged, insofar as *laissez-faire* equilibrium quantities of carbon-sink enhancing goods should be provided in lower than optimal quantities.

[4] This representation relies on a simplified description of the climate system, whereby change in the radiative budget and average surface temperature are linearly linked (Intergovernmental Panel on Climate Change, 2001).

0 and $D''(.) \geq 0$). Climate change economic damages are assumed to be a 'pure' global externality. Agents thus do not spontaneously internalize the effect on climate that their economic decisions induce. In addition, we will assume, that climate-change related damage causes an economic loss on agents that additively reduces the total surplus. The discount rate is denoted by δ. The problem faced by a (risk-neutral) social planner who intends to maximize the welfare over a (possibly infinite) time horizon T case can be written as follows:

$$\max_{\mathbf{x}_t} \int_0^T [S(\mathbf{x}_t) - D(\theta(\mathbf{z}_t))]e^{-\delta t} dt \tag{3}$$

The (current) Hamiltonian of the problem and the optimality conditions are given by (the time index t is implicit and omitted):

$$\mathcal{H} = S(\mathbf{x}_t) - D(\theta(\mathbf{z}_t)) - \sum_{j=1}^{n} \lambda_j(-\tau_{jt} z_{jt} + \varepsilon_j(\mathbf{x}_t)) \tag{4a}$$

$$\mathbf{x} \in \arg\max_{\mathbf{x}} \mathcal{H} \tag{4b}$$

$$\dot{\lambda}_j = \delta\lambda_j + \frac{\partial \mathcal{H}}{\partial z_j} \quad (j = 1, \ldots, n) \tag{4c}$$

Note that the expressions of the Hamiltonian (4a) and of the equations of motion of shadow prices (4c) are modified compared to their canonic expression in such a way that all λ_j are positive. In fact, the state variable here (concentrations) has a negative impact on the objective function. The co-state variable attached to the equation of motion of z_j should thus also be negative. We thus choose λ_j here as the opposite of the standard shadow price.

Let $\mathbf{H}_{[S,\mathbf{x}]}$ and $\mathbf{H}_{[\varepsilon_j,\mathbf{x}]}$ be the $m \times m$-Hessian matrices of S and ε_j, respectively. The concavity of the Hamiltonian with respect to the command variable $\mathbf{x}$ is ensured by the fact that the Hessian matrix of $\mathcal{H}$ with respect to $\mathbf{x}$ is definite negative. Differentiating twice $\mathcal{H}$ with respect to $\mathbf{x}$, we have:

$$\mathbf{H}_{[\mathcal{H},\mathbf{x}]} = \mathbf{H}_{[S,\mathbf{x}]} - \sum_{j=1}^{n} \lambda_j \mathbf{H}_{[\varepsilon_j,\mathbf{x}]} \tag{5}$$

Hence, we need the matrix $\left(\mathbf{H}_{[S,\mathbf{x}]} - \sum_{j=1}^{n} \lambda_j \mathbf{H}_{[\varepsilon_j,\mathbf{x}]}\right)$ to be non-singular and definite negative. As discussed above, all λ_j are non-negative at the optimum. Therefore, standard assumptions on the concavity of $S(.)$ and $\varepsilon_j(.)$ ($\mathbf{H}_{[S,\mathbf{x}]}$ definite negative and $\mathbf{H}_{[\varepsilon_j,\mathbf{x}]}$ semi-definite positive) are sufficient to ensure that the Hamiltonian is concave with respect to the command variable. Note also that in the case of a linear relationship between emissions and $\mathbf{x}$, the second term in equation (5) is zero. The necessary conditions, whereby all derivatives of $\mathcal{H}$ with respect to the command variable are zero, are thus also sufficient to respect the static optimality condition (4b):

$$\phi_i(\boldsymbol{\lambda}, \mathbf{x}) = S'_{x_i}(\mathbf{x}) - \sum_{j=1}^{n} \lambda_j \varepsilon'_{j,x_i}(\mathbf{x}) = 0 \quad (i = 1, \ldots, m) \tag{6}$$

Equation (6) implies that, at each point in time, the marginal surplus drawn from good i is equal to the sum of marginal emission content of x_i in all gases weighted by the corresponding shadow prices. At the optimum, λ_j thus reflects the marginal social value of emissions in gas j.

From equation (6), it is possible to locally define a vector-valued function $\mathbf{f}(.)$ such that $\mathbf{x}^* = \mathbf{f}(\boldsymbol{\lambda})$ in the neighborhood of $\boldsymbol{\lambda}^*$ ($\boldsymbol{\lambda}^*$ is the solution of (4b)–(4c)). Hence, $\mathbf{f}(.)$ implicitly defines the optimal levels of consumption in all goods as a function of their social value. In order to use the implicit function theorem, we need the Jacobian of $\boldsymbol{\phi}$ to be non-singular in λ^*. Notice that $\mathbf{J}_{[\boldsymbol{\phi},\mathbf{x}]}(\boldsymbol{\lambda}^*) = \mathbf{H}_{[S,\mathbf{x}]} - \sum_{j=1}^{n} \lambda_j \mathbf{H}_{[\varepsilon_j,\mathbf{x}]}$, which is non singular by assumption (see above). $\mathbf{J}_{[\boldsymbol{\phi},\mathbf{x}]}$ thus represents the marginal change in consumption levels as a function of the shadow prices. It is obtained by totally differentiating (6) with respect to $\mathbf{x}$ and $\boldsymbol{\lambda}$:

$$\mathbf{J}_{[\mathbf{f},\boldsymbol{\lambda}]}(\boldsymbol{\lambda}^*) = \left(\mathbf{H}_{[S,\mathbf{x}]}(\mathbf{x}^*) - \sum_{j=1}^{n} \lambda_j^* \mathbf{H}_{[\varepsilon_j,\mathbf{x}]}(\mathbf{x}^*)\right)^{-1} \cdot \mathbf{J}^T_{[\boldsymbol{\varepsilon},\mathbf{x}]}(\mathbf{x}^*) \qquad \left[= \mathbf{H}^{-1}_{[S,\mathbf{x}]}(\mathbf{x}^*) \cdot \mathbf{E}^T\right] \tag{7}$$

In the general case of a non-linear relationship between emissions and equilibrium quantities $\mathbf{x}$, the marginal effect of a change in the shadow prices is twofold: (i) change in quantities in $\mathbf{x}$, which is adjusted in order to maximize $S(.)$ according to $\mathbf{H}_{[S,\mathbf{x}]}$; and (ii) marginal emission content of each good is modified as $\mathbf{x}$ changes, and these changes have also to be valued at the corresponding shadow prices. This two effects are embedded in the matrix in parenthesis. In the case of a linear relationship between emissions and $\mathbf{x}$, the latter effect vanishes as the marginal content of each good is constant. The expression in square brackets in equation (7) thus gives the Jacobian of $\mathbf{f}$ in this case. Note that the m-vector $\mathbf{J}^T_{[\boldsymbol{\varepsilon},\mathbf{x}]}(\mathbf{x}^*) \cdot \boldsymbol{\lambda}$ represents the marginal emission profile evaluated at the price $\boldsymbol{\lambda}$. The i-th entry of this vector is the total social value of emissions caused by good i at the margin.

$\mathbf{f}(\boldsymbol{\lambda})$ represents the 'demand' in private goods as a function of the 'prices' of greenhouse gases. Marginal abatement costs could thus be derived from equation (7) as the marginal loss in private surplus resulting from a change in consumption levels and have to be compared with the marginal social value of GHGs. Unlike cost-minimizing models (Kandlikar, 1996; Moslener and Requate, 2001), the formulation presented here is more general and, in particular, does not require any specific assumption about abatement costs separability. Note that $\mathbf{f}(\boldsymbol{\lambda})$ depends on the marginal substitution rates between all goods.

Replacing $\mathbf{x}$ by $f(\boldsymbol{\lambda})$ in equations (4c), and combining them with equations (2) and initial and final conditions define the dynamics of shadow prices and concentrations.

$$\dot{z}_j = -\tau_j z_j + \varepsilon_j(\mathbf{f}(\boldsymbol{\lambda})) \quad (j = 1, \ldots, n) \tag{8a}$$

$$\dot{\lambda}_j = (\delta + \tau_j)\lambda_j - D'(\theta(\mathbf{z}))\theta'_{z_j}(\mathbf{z}) \quad (j = 1, \ldots, n) \tag{8b}$$

4. A simple illustration of the (non-)equivalence between GWP and optimal shadow prices ratio

Consider first the case of linear economic damage **and** linear change in temperature with respect to the concentration in at least one gas j. Equation (8b) for that particular gas j is a first-order linear differential equation with a constant, positive right-hand-side. We thus have the following result:

PROPOSITION 1. *Consider a gas j with a constant decay rate. Assume that the temperature change is linear with respect to the concentration in gas j. That is, $\theta'_{z_j}(\mathbf{z}) = \theta_j$. If the economic valuation of the economic damage is linear with respect to the temperature change ($D(\theta) = \alpha\theta$) then the optimal shadow price for gas j (for an infinite planning horizon) is :*

$$\lambda_{jt} = \frac{\alpha.\theta_j}{\delta + \tau_j} \tag{9}$$

Proof. The general solution of equation (8b) under the conditions stated in the proposition is $\lambda_{jt} = \frac{\alpha.\theta_j}{\delta+\tau_j} + k_j.e^{(\delta+\tau_j)t}$. The transversality condition gives : $\lim_{t\to+\infty} \lambda_{jt}.e^{-\delta t} = 0$, which imposes $k_j = 0$. QED.

Note that in this particular case, λ_j is constant over time. Even under the very restrictive conditions of the proposition, this result sheds light on two of the major often cited drawbacks of the GWP concept. Consider that the conditions hold for two gases j and k ($D' = \alpha\theta(\mathbf{z})$, $\theta'_{z_j} = \theta_j$, and $\theta'_{z_k} = \theta_k$), then the shadow prices ratio and GWP (1) become:

$$\frac{\lambda_j}{\lambda_k} = \frac{\theta_j}{\theta_k}\frac{\delta + \tau_k}{\delta + \tau_j} \text{ and } \mathrm{GWP}_{j,k}(\hat{T}) = \frac{\theta_j\tau_k}{\theta_k\tau_j}\frac{1 - e^{-\tau_j\hat{T}}}{1 - e^{-\tau_k\hat{T}}}, \; \lim_{\hat{T}\to+\infty} \mathrm{GWP}_{j,k}(\hat{T}) = \frac{\theta_j\tau_k}{\theta_k\tau_j} \tag{10}$$

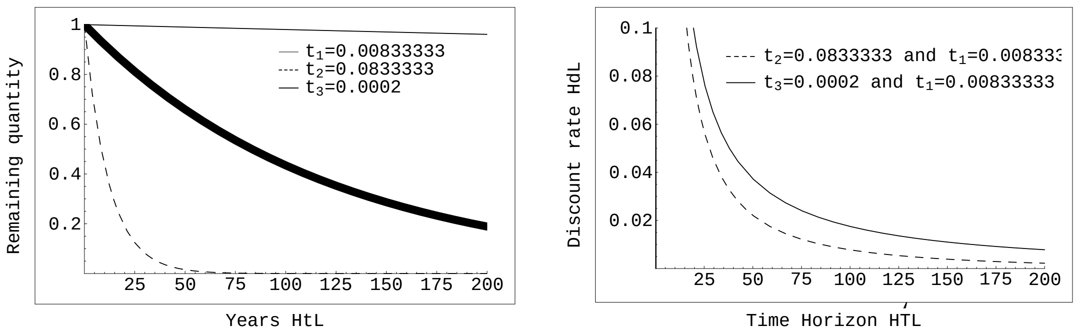
From equation (10), it is easily seen that the GWP differs from the ratio of optimal shadow prices for at least two reasons: (i) the chosen time-horizon ($\hat{T}$) which appears in the expression of the GWP, but not in the shadow price ratio, and (ii) the discount rate (δ) that only affects the shadow price ratio.

Moreover, the difference between the GWP and the ratio of optimal shadow prices only depends on the natural decay rates (τ_j and τ_k), the discount rate (δ) and the time-horizon used in the computation of the GWP ($\hat{T}$). As a matter of fact, for any pair of gases, it is possible to find a combination of δ and $\hat{T}$ such that the GWP is equal to the ratio of optimal shadow prices. This property is summarized in the following proposition.

PROPOSITION 2. *In general, the Global Warming Potential index, given any two gases j and k, is not equal to the ratio of the optimal shadow prices associated with these two gases. However, under the conditions of proposition 2, there exists a unique time-horizon $\hat{T}$ used in the computation of the GWP such that this equality is fulfilled for any given 2-uple of gases j, k.*

Proof. It is sufficient to see that the function $\delta(\hat{T})$, implicitely defined by the equation below, is monotically decreasing with respect to $\hat{T}$ from $+\infty$ to 0:

$$\frac{\delta + \tau_k}{\delta + \tau_j} = \frac{\tau_k}{\tau_j} \frac{1 - e^{-\tau_j \hat{T}}}{1 - e^{-\tau_k \hat{T}}} \tag{11}$$



a. Evolution of concentration for three illustrative constant decay rates

b. Combinations of $(\hat{T}, \delta)$ for which $\mathrm{GWP}_{j,1} = \frac{\lambda_j}{\lambda_1}$

Figure 2. Trade-off between discount rate and time-horizon for linear damage and linear temperature response

The result of proposition 2 is illustrated on Figure 2. On the left hand side of Figure 2, the evolution of gases is shown for three constant decay rates. Three characteristic lifetimes are examined: 120 (thick line), 12 (dashed), and 5,000 (solid) years. These characteristic lifetimes correspond to values of decay rates of $\tau_1 = 0.0083$, $\tau_2 = 0.0833$, and $\tau_3 = 0.0002$, respectively. The first two can be associated with characteristic lifetimes of CO_2[5] and CH_4, respectively. Some trace gases such as substitutes to CFC are characterized with very long atmospheric

[5] Indeed, the behavior of CO_2 in the atmosphere is much more complex and cannot be satisfactorily summarized through a constant decay rate, as the 'speed' of exchanges between different carbon pools (terrestrial sinks, upper oceans, deep oceans, atmospheric) varies widely. More sophisticated approaches approximate the dynamics of carbon as a sum of exponential (Bradford and Keller, 2000). This however does not affect the general conclusions drawn in this section.

residence times of a magnitude similar to the one implied by τ_3. For illustrative purposes, the chosen range of atmospheric lifetime is deliberately very wide. After 200 years, the remaining fraction of a unitary emission pulse of gas 1 is almost unnoticeable (order of magnitude 10^{-8}), while it is approximately 0.96 for gas 3.

The graph on the right hand side illustrates the trade-off between the time-horizon $\hat{T}$ and the discount rate, with gas 1 taken as the reference. If $\hat{T}$ is fixed, for example at 100 years (the commonly used convention taken in the IPCC), then it is possible to find a value of the discount rate such that $\mathrm{GWP}_{2,1}(100) = \frac{\lambda_2}{\lambda_1}$. In this case, this value would be approximately 2.21%. For a time horizon of 500 years, the value of the discount rate that ensures equality for the GWP of gas 2 and the optimal shadow price ratio drops to 0.015%. As for gas 3, the characteristic values of the discount rate are higher (3.73% and 0.249% for 100 and 500 years, respectively). Since gas 3 is longer-lived than gas 2, a higher discount rate is needed to ensure the equality between the GWP and the shadow price ratio. It also follows that longer time-horizons imply lower characteristic values of the discount rate, these values also being closer for any pair of gases. However, one sees from Figure 2 that as soon as the number of gases is greater than two, there exists no 2-uple $(\delta, \hat{T})$ such that the *all* GWPs could be equal to the corresponding shadow price ratio.

Conversely, one could set the discount rate and derive the equivalent time-horizon. For commonly used values of the discount rates in long-term environmental issues, for example lower than 3%, the time-horizons such that the GWP is equal to the optimal shadow price ratio appear reasonably close to the 100-year mark. For instance, for a discount rate of 1.5%, $\hat{T}$ ranges from 65.3 to 114.7 years, for gas 2 and 3, respectively. However, a change in $\hat{T}$ may impact significantly the value of the GWP. Computing the GWP of gas 2 (resp. 3) over a time-horizon of 65.3 (resp 114.7) years instead of 100 indeed yields a 34.2% (resp. 5.2%) increase in the index. In addition, for very low discount rates sometimes advocated in the case of climate change, the difference in "equivalent" time-horizons widens between short- and long-lived GHG.

What does this have to say about the validity of the GWP as a climate change index? First, this confirms the fact that the GWP is not an adequate metric whenever it is used to measure the *economic* equivalence between gases. This result is in line with findings from Kandlikar (1996), but provides a broader perspective: in his paper, Kandlikar assumes that the time-horizon of the social planner's program is the same as the one used in the computation of the GWP. The linear case examined here greatly simplifies the links between radiative forcing and economic damage. This supposedly reduces the bias between radiative-based and welfare-based indices. Notwithstanding, a bias remains, rooting in the lack of discounting and the arbitrariness in the choice of the time-horizon.

Second, the results highlight the possible trade-off between the discount rate and the time-horizon chosen in the computation of the GWP. One could argue that the choice of the discount rate is not more arbitrary than the choice of $\hat{T}$. The value of the discount rate has prompted fierce debates among economists, especially for long-term environmental issues such as climate change. The choice of any value for δ is *inter alia* contingent to assumptions about future preferences and productivity of capital. From figure 2, the 100-year assumption might appear as a compromise that leads to reasonable implicit values of the discount rate. However, as indicated by the computations above, this can induce significant distortions in the GHG equivalence rule, in particular for short-lived gases.

5. Dynamics of the economic-ecological system

5.1. Linearization and steady-state

Let us now consider a more general form of the system (8a)–(8b). Let $\mathbf{\Delta}_{[-\tau]}$ and $\mathbf{\Delta}_{[\delta+\tau]}$ be the $n \times n$-diagonal matrices with $(-\tau_j)$ and $(\delta + \tau_j)$ on the j-th component of the first diagonal, respectively. By linearizing the system through the Taylor's development of (8a)–(8b) in the neighborhood of any point $(\bar{\mathbf{z}}, \bar{\boldsymbol{\lambda}})$, one obtains the following 2-n system written in matrix form:

$$\begin{pmatrix} \dot{\mathbf{z}} \\ \dot{\boldsymbol{\lambda}} \end{pmatrix} \approx \begin{pmatrix} \mathbf{\Delta}_{[-\tau]} & \mathbf{A}(\bar{\boldsymbol{\lambda}}) \\ \mathbf{B}(\bar{\mathbf{z}}) & \mathbf{\Delta}_{[\delta+\tau]} \end{pmatrix} \cdot \begin{pmatrix} \mathbf{z} \\ \boldsymbol{\lambda} \end{pmatrix} + \begin{pmatrix} \mathbf{a}(\bar{\boldsymbol{\lambda}}) \\ \mathbf{b}(\bar{\mathbf{z}}) \end{pmatrix} \tag{12}$$

where

$$\mathbf{A}(\bar{\boldsymbol{\lambda}}) = \mathbf{J}_{[\boldsymbol{\varepsilon},\mathbf{x}]}(\mathbf{f}(\bar{\boldsymbol{\lambda}})) \cdot \mathbf{J}_{[\mathbf{f},\boldsymbol{\lambda}]}(\bar{\boldsymbol{\lambda}}) \qquad \left[= \mathbf{E} \cdot \mathbf{H}^{-1}_{[S,\mathbf{x}]}(\mathbf{x}^*) \cdot \mathbf{E}^T\right] \tag{13a}$$

$$\mathbf{B}(\bar{\mathbf{z}}) = -D''(\theta(\bar{\mathbf{z}})).\boldsymbol{\nabla}_{[\theta,\mathbf{z}]}(\bar{\mathbf{z}}) \cdot \boldsymbol{\nabla}^T_{[\theta,\mathbf{z}]}(\bar{\mathbf{z}}) - D'(\theta(\bar{\mathbf{z}})).\mathbf{H}_{[\theta,\mathbf{z}]}(\bar{\mathbf{z}}) \tag{13b}$$

Again, the part of equation (13a) in square brackets gives the expression of $\mathbf{A}$ in the particular case of constant emission factors. $\mathbf{a}(\bar{\boldsymbol{\lambda}})$ and $\mathbf{b}(\bar{\mathbf{z}})$ are two n-vectors independent of $\boldsymbol{\lambda}$ and $\mathbf{z}$. $\mathbf{A}(\bar{\boldsymbol{\lambda}})$ reflects the optimal change in emissions resulting from a marginal change in the shadow prices, while $\mathbf{B}(\bar{\mathbf{z}})$ represents the marginal change in damage caused by a small variation of the concentrations. Note that if the damage and the temperature are linear, $\mathbf{B}$ is zero and we find back the diagonal sub-system in $\boldsymbol{\lambda}$ discussed above.

In the general case of non-constant entries of matrices $\mathbf{A}(\bar{\boldsymbol{\lambda}})$ and $\mathbf{B}(\bar{\mathbf{z}})$, the steady state, which is defined by $\dot{\mathbf{z}} = 0$ and $\dot{\boldsymbol{\lambda}} = 0$, can be iteratively computed. In general, the evolution of the system is discussed by the analysis of the eigen-values of the matrix that appears in equation (12).

Figure 3 shows the projection of the phase diagram in the space (z_j, λ_j) (holding (z_{-j}, λ_{-j}) constant) in the case of non-linear damage and/or radiative forcing functions. Graphical analysis of the trajectories shows that the steady-state is a saddle-point. Therefore, there are only two optimal paths converging to the steady-state $(z_j^\infty, \lambda_j^\infty)$. These paths are depicted in solid lines in Figure 3. Depending on initial conditions, the shadow price is thus monotonically decreasing or increasing over time.

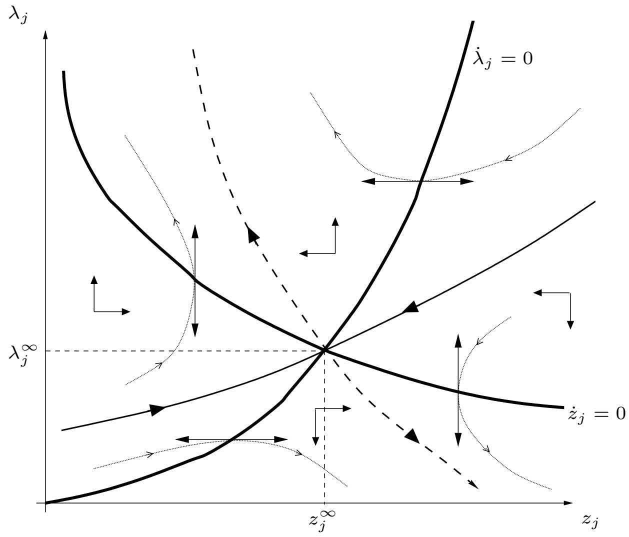


Figure 3. Phase diagram for z_j, λ_j with $(\mathbf{z}_{-j}, \boldsymbol{\lambda}_{-j})$ held constant

However, as the isocline vary with $\mathbf{z}_{-j}$ and $\boldsymbol{\lambda}_{-j}$, it is not possible to extend those results to the general case. Indeed, as shown by Moslener and Requate (2001) non-monotonic optimal paths are possible. The following section further investigates the evolution of shadow prices.

The computation of the steady-state is easier if the components of $\mathbf{A}(\bar{\boldsymbol{\lambda}})$ and $\mathbf{B}(\bar{\mathbf{z}})$ are constant. We can then use the linearized system (12) to directly compute the steady-state. From equations (13a)–(13b), one sees that sufficient conditions for the two matrices not to depend on $\mathbf{z}$ and $\boldsymbol{\lambda}$ are: (i) constant emission factors, (ii) $S(.)$ as a linear or quadratic form (or a combination of both) of $\mathbf{x}$, (iii) $D(.)$ quadratic with respect to θ ($D(\theta) = \frac{1}{2}\beta\theta^2$), (iii) $\theta(.)$ linear with respect to $\mathbf{z}$. Note also that in the case of linear temperature response to concentrations, the second term in $\mathbf{B}(\bar{\boldsymbol{\lambda}})$ is zero. If in addition, $D(.)$ is quadratic, $\mathbf{b}(\bar{\boldsymbol{\lambda}})$ is also zero. The steady state can thus be computed as:

$$\begin{pmatrix} \mathbf{z}^\infty \\ \boldsymbol{\lambda}^\infty \end{pmatrix} = \begin{pmatrix} \boldsymbol{\Delta}_{[-\tau]} & \mathbf{E} \cdot \mathbf{H}^{-1}_{[S,\mathbf{x}]} \cdot \mathbf{E}^T \\ -\beta \boldsymbol{\nabla}_{[\theta,\mathbf{z}]} \cdot \boldsymbol{\nabla}^T_{[\theta,\mathbf{z}]} & \boldsymbol{\Delta}_{[\delta+\tau]} \end{pmatrix}^{-1} \cdot \begin{pmatrix} \mathbf{a} \\ 0 \end{pmatrix} \tag{14}$$

5.2. Shadow prices dynamics

From the equation of motion for λ_j (8b), it can be easily seen that, at the optimum, λ_j represents the integral over time of the flows of marginal damages caused by gas j, discounted by the discount rate (δ) and the absorption rate (τ_j). Hence, together with equation (6), this equation translates the standard static result condition, whereby the marginal social value of emissions should be equal to the marginal damage, in a dynamic setting.

Consider now the ratio of the shadow prices of two arbitrarily chosen gases j and k. Differentiating the ratio with respect to time and using equation (8b), one obtains:

$$\dot{\left(\frac{\lambda_j}{\lambda_k}\right)} = \frac{\lambda_j}{\lambda_k}(\tau_j - \tau_k) + \frac{D'(\theta(\mathbf{z}))\theta'_{z_k}(\mathbf{z})}{\lambda_k}\left(\frac{\lambda_j}{\lambda_k} - \frac{\theta'_{z_j}(\mathbf{z})}{\theta'_{z_k}(\mathbf{z})}\right) \tag{15}$$

The evolution over time is governed by two important quantities: (i) the difference in natural absorption rates ($\tau_j - \tau_k$), and (ii) the ratio of average impact on temperature ($\theta'_{z_j}(\mathbf{z})/\theta'_{z_k}(\mathbf{z})$). Note that, in particular, an obvious solution to have a constant ratio of shadow price is that both gases have exactly the same lifetime and the same marginal impact on temperature. Another solution is, as found in the previous section, that economic damage is linear with respect to temperature change, and θ'_{z_j} and θ'_{z_k} are constant.

Now, assume, without loss of generality, that $\tau_{jt} \geq \tau_{kt}$ holds for all t. That is, gas j is shorter-lived than gas k. Since λ_j and λ_k are positive, the first term in equation (15) is positive. Economic valuation of the damage is assumed to be increasing with respect to the average impact on temperature, and the marginal impact of concentration in gas k on temperature is positive. Therefore, the positivity of the second term in equation (15) depends on the relative position of shadow prices and radiative forcing ratios. If $\theta'_{z_j}(\mathbf{z})/\theta'_{z_k}(\mathbf{z})$ is sufficiently low (gas j, in addition to be shorter-lived than gas k, has a sufficiently lower direct impact on temperature), λ_j/λ_k is increasing over time. In this case, if at any time λ_j/λ_k is greater than $\theta'_{z_j}(\mathbf{z})/\theta'_{z_k}(\mathbf{z})$, then λ_j/λ_k is increasing over time. From equation (15), one also sees that if natural absorption rates are identical, the evolution of the shadow price ratio is only driven by the relative position of λ_j/λ_k with $\theta'_{z_j}(\mathbf{z})/\theta'_{z_k}(\mathbf{z})$.

6. Decentralizing the optimum through multi-gas price instruments

6.1. First-best multi-gas price instruments

In the problem described in (3), each gas is accounted for separately. Hence, in this case, there is no specific need to use a metric to compare or aggregate GHGs. Imagine that multi-gas targets are adopted and that any multi-gas economic instruments that are to be used have to rely on this metric. The first question is: by using such instruments, is it possible to decentralize the economic optimum given by equations (4b)–(4c)?

Let p_t be the emission tax expressed in dollars per ton of gas 1-equivalent (e.g., gas 1 is CO_2), and γ_j the conversion coefficient of gas j into gas 1. Total emissions in tons of gas 1-equivalent are $\sum_{j=1}^{n} \gamma_j \varepsilon_j(\mathbf{x})$. Agents maximize the private part of the surplus. This thus leads to the following conditions:

$$S'_{x_i}(\mathbf{x}_t) = p_t \sum_{j=1}^{n} \gamma_j \varepsilon'_{x_i,j}(\mathbf{x}) \quad (i = 1, \ldots, m) \tag{16}$$

At the optimum, consumption levels are such that the tax is equal to the ratio of the marginal (private) surplus of x_i and the marginal emission content of consumption of x_i expressed in tons of gas 1-equivalent. By identifying p and γ_j in equation (4b), the emission tax and equivalence coefficients that yield the optimal emissions in each gas are (normalization in gas 1):

$$p = -\lambda_1^* \text{ and } \gamma_j = \frac{\lambda_j^*}{\lambda_1^*} \quad j = 1, \ldots, n \tag{17}$$

where λ_j^* are the solutions of the system (8a)–(8b). As argued by Kandlikar (1996), the conversion from any gas j into gas 1 should be based on the ratio of the shadow prices of the two gases. As shown in equation (15), γ_j should thus depend on the ratio of radiative forcing between gas j and gas 1, on the difference of the atmospheric lifetime, on the marginal value of economic damages, and on the discount rate (though λ_1). In general, the metric is not constant over time.

6.2. Second-best multi-gas price instruments

Assume now that a metric has be agreed upon that allows to convert emissions in any gas j into gas 1-equivalent. Therefore, the social planner cannot use this metric as a command variable. Assume that the use of this metric has been made mandatory, hence becoming a parameter in the optimal control problem. That means that the only command variable left to the social planner is the tax p_t.

First, using a similar argument as above, one can implicitly define the m-vector of consumption as a function of the vector of the tax on all gases in gas 1-equivalents $(p_t \cdot \boldsymbol{\gamma})$:

$$\tilde{\mathbf{x}} = \mathbf{g}(p_t\gamma) \tag{18}$$

which, through the total differentiation of (16), leads to an expression similar to equation (7).

The problem faced by the social planner is modified and becomes:

$$\max_{p_t} \quad \int_0^T \left[S(\mathbf{g}(p_t\boldsymbol{\gamma})) - D(\theta(\mathbf{z}))\right] e^{-\delta t} dt \tag{19a}$$

$$\text{st} \quad \dot{z}_{jt} = -\tau_{jt} z_{jt} + \varepsilon_j(\mathbf{g}(p_t\boldsymbol{\gamma})) \quad (j = 1, \ldots, n) \tag{19b}$$

The (current) Hamiltonian of the modified problem is given by:

$$\tilde{\mathcal{H}} = S(\mathbf{g}(p\boldsymbol{\gamma})) - D(\theta(\mathbf{z})) - \sum_{j=1}^{n} \mu_j(-\tau_j z_j + \varepsilon_j(\mathbf{g}(p\boldsymbol{\gamma}))) \tag{20}$$

and the necessary conditions of optimality are (the time index t is implicit and omitted):

$$p_t \in \arg\max_{p_t} \tilde{\mathcal{H}} \tag{21a}$$

$$\dot{\mu}_j = \delta\mu_j + \frac{\partial \mathcal{H}}{\partial z_j} \quad (j = 1, \ldots, n) \tag{21b}$$

where $\boldsymbol{\mu} = (\mu_j)_{j=i,\ldots,n}$ are the new shadow prices attached to equations of motion of the state variable in the modified problem.

Differentiating $\tilde{\mathcal{H}}$ with respect to p_t yields at the optimum:

$$\frac{\partial \tilde{\mathcal{H}}}{\partial p} = \boldsymbol{\nabla}_{[S,\mathbf{x}]}(g(p\gamma)) \cdot \mathbf{J}_{[\mathbf{g},\boldsymbol{\lambda}]}(p\gamma) \cdot \gamma - \boldsymbol{\mu}^T \cdot \mathbf{J}_{[\varepsilon,\mathbf{x}]}(\mathbf{g}(p\gamma)) \cdot \mathbf{J}_{[\mathbf{g},\boldsymbol{\lambda}]}(p\gamma) \cdot \boldsymbol{\gamma} = 0 \tag{22}$$

Introducing equation (16) in equation (22) and re-arranging, one obtains:

$$p_t = \frac{\boldsymbol{\gamma}^T \cdot \left(\mathbf{J}_{[\boldsymbol{\varepsilon},\mathbf{x}]}(\mathbf{g}(p\boldsymbol{\gamma})) \cdot \mathbf{J}_{[\mathbf{g},p\boldsymbol{\gamma}]}(p\boldsymbol{\gamma})\right) \cdot \boldsymbol{\mu}}{\boldsymbol{\gamma}^T \cdot \left(\mathbf{J}_{[\boldsymbol{\varepsilon},\mathbf{x}]}(\mathbf{g}(p\boldsymbol{\gamma})) \cdot \mathbf{J}_{[\mathbf{g},p\boldsymbol{\gamma}]}(p\boldsymbol{\gamma})\right) \cdot \boldsymbol{\gamma}} \tag{23}$$

Equation (23) gives the general expression of the second-best tax. For a given $\boldsymbol{\gamma}$, the optimal tax is defined a linear combination of the optimal values of the shadow prices $\boldsymbol{\mu}$ in the modified problem. Recall that $\left(\mathbf{J}_{[\boldsymbol{\varepsilon},\mathbf{x}]} \cdot \mathbf{J}_{[\mathbf{g},p\boldsymbol{\gamma}]}\right)$ represents the true marginal change in emissions consecutively to a change in the social value of the greenhouse gas prices. Therefore $\boldsymbol{\gamma}^T \cdot \left(\mathbf{J}_{[\boldsymbol{\varepsilon},\mathbf{x}]} \cdot \mathbf{J}_{[\mathbf{g},p\boldsymbol{\gamma}]}\right)$ represents the aggregation of this change in emissions according to the $\boldsymbol{\gamma}$ equivalent rule. as an illustration, consider that the entries of $\boldsymbol{\gamma}$ are the standard GWP. In this case, $\boldsymbol{\gamma}^T \cdot \left(\mathbf{J}_{[\boldsymbol{\varepsilon},\mathbf{x}]} \cdot \mathbf{J}_{[\mathbf{g},p\boldsymbol{\gamma}]}\right)$ the marginal change in $\mathbf{CO_2}$**-equivalent** emissions. Given this, the expression of the numerator in (23) is now easier to interpret. The optimality condition (21a) implies that the optimal level of the second-best tax should be proportional to the marginal change in CO_2-equivalent emissions evaluated at the current corresponding shadow prices $\boldsymbol{\mu}$. The denominator can be interpreted as a normalization factor. It also represents the marginal change in CO_2-equivalent emissions, but evaluated through the given equivalence rule $\boldsymbol{\gamma}$. The fact that there appears a normalization factor in the expression p_t should come as no surprise. Indeed, $\boldsymbol{\gamma}$ is defined as relative equivalence rule. That means that it should be always possible to change the reference gas (from carbon dioxide to methane for instance) without changing the optimal solution of the program (21a)–(21b). Obviously, this implies a corresponding change in the value of p_t (multiplied by the norm of $\boldsymbol{\gamma}$ according to $\mathbf{J}_{[\boldsymbol{\varepsilon},\mathbf{x}]} \cdot \mathbf{J}_{[\mathbf{g},p\boldsymbol{\gamma}]}$).

In the particular case (constant emission factors, quadratic-linear surplus, quadratic damage, linear temperature response) used in equation (27), the optimal tax can be rewritten as follows:

$$p_t = \frac{\boldsymbol{\gamma}^T \cdot \mathbf{A} \cdot \boldsymbol{\mu}}{\boldsymbol{\gamma}^T \cdot \mathbf{A} \cdot \boldsymbol{\gamma}} = \frac{(\mathbf{E}^T\boldsymbol{\gamma})^T \cdot \mathbf{H}^{-1}_{[S,\mathbf{x}]} \cdot (\mathbf{E}^T\boldsymbol{\mu})}{(\mathbf{E}^T\boldsymbol{\gamma})^T \cdot \mathbf{H}^{-1}_{[S,\mathbf{x}]} \cdot (\mathbf{E}^T\boldsymbol{\gamma})} \tag{24}$$

From equation (24), one sees the importance of the substitutions in the consumption of private goods, and which are embedded in the matrix $\mathbf{H}^{-1}_{[S,\mathbf{x}]}$. Notice that $\mathbf{E}^T \cdot \boldsymbol{\gamma}$ is the emission profile for $\mathbf{x}$ using the $\boldsymbol{\gamma}$-equivalence rule. That is, the i-th component of the m-vector $\mathbf{E}^T \cdot \boldsymbol{\gamma}$ is the total CO_2 equivalent emissions related to consumption of good i. If $\boldsymbol{\gamma}$ and the entries of $\mathbf{H}^{-1}_{[S,\mathbf{x}]}$ are constant over time, then the rate of change of the optimal tax over time is obtained as the linear combination of the rates of the change of $\boldsymbol{\mu}$ over time.

$$\dot{p}_t = \frac{\boldsymbol{\gamma}^T \cdot \mathbf{A} \cdot \dot{\boldsymbol{\mu}}}{\boldsymbol{\gamma}^T \cdot \mathbf{A} \cdot \boldsymbol{\gamma}} \tag{25}$$

If the stricter conditions examined in section 4 hold, that is damage is linear and $\mathbf{B} = 0$, we know from the analysis conducted in section 4 that all components of $\boldsymbol{\mu}$ are constant and equal to $\frac{\alpha\theta_j}{\delta+\tau_j}$. In this case, the second-best GWP-based tax is thus also constant over time. It is thus equal to a linear combination of radiative forcings of all gases, weighted by the sum of the discount rate and their decay rates. The coefficients in this linear combination depend on the coefficients in $\boldsymbol{\gamma}$, on the marginal substitution rates between all goods, and on the emission factors.

The linear assumption –although very simplifying– provides an intuitive insight at an important property of the second-best optimal tax. Section 4 has shown that, in general, GWPs were not equivalent to the optimal shadow price ratios. Furthermore, in the case of more than two gases –and there are six gases listed in the Kyoto-basket– it was not possible to change the chosen time-horizon in order to ensure this equality. By construction, the GWP-based tax should lead to consumption levels different to the first-best optimum derived from equations (8a)-(8b) to lower welfare values. However, the conjecture is the tax makes it possible to reach higher global welfare values than in the case of tax solely based on $\boldsymbol{\gamma}$ ($p_t = \boldsymbol{\gamma}^T \cdot \mathbf{A}\boldsymbol{\gamma}$).

Let now return to the general formulation of the problem. Using $\mathbf{g}(p\boldsymbol{\gamma})$, one can re-write the system can be rewritten as follows:

$$\dot{z}_j = -\tau_j z_j + \varepsilon_j(\mathbf{g}(p)) \quad (j = 1, \ldots, n) \tag{26a}$$

$$\dot{\mu}_j = (\delta + \tau_j)\mu_j - D'(\theta(\mathbf{z}))\theta'_{z_j}(\mathbf{z}) \quad (j = 1, \ldots, n) \tag{26b}$$

Following the same reasoning, the modified differential system can be solved through the linearization around the steady-state and finding the eigenvalues of the system written in matrix form. For the sake of simplicity, we assume here that the assumptions made at the end of section 5.1 ($\mathbf{A}$ and $\mathbf{B}$ do not depend on $\mathbf{z}$ or $\boldsymbol{\lambda}$). Therefore, we have the following modified system:

$$\begin{pmatrix} \mathbf{z}^\infty \\ \boldsymbol{\lambda}^\infty \end{pmatrix} = \begin{pmatrix} \boldsymbol{\Delta}_{[-\tau]} & \mathbf{A} \cdot \boldsymbol{\gamma}^T \boldsymbol{\gamma} \cdot \mathbf{A} \\ \mathbf{B} & \boldsymbol{\Delta}_{[\delta+\tau]} \end{pmatrix}^{-1} \cdot \begin{pmatrix} \tilde{\mathbf{a}} \\ 0 \end{pmatrix} \tag{27}$$

If the matrix in parenthesis is non-singular, then there exists a unique steady-state in the modified system. This is steady-state under the GWP-based tax regime. At this level of generality, it is difficult to be more conclusive about the dynamic of the system without taking more specific assumptions about the functions, in particular with respect to $S(.)$. Solving for $\boldsymbol{\mu}$ in the general case should lead to μ_i that vary over time according to the changes in the marginal damages. The conjecture is that the changes in the GWP-based tax defined in (24) should be mitigated through the linear combination with constant γ_i.

7. Concluding remarks

To the question: “What would be the optimal equivalence rule between various greenhouse gases, which differ in their atmospheric lifetimes and climate impacts?”, the economic answer is a welfare-based index that balances marginal economic damages, marginal abatement costs, and differentiates between short- and long-term for instance through discounting long-term damages. Propositions of this nature can be found in the economic literature, and the purpose of this paper was not to produce a new one. However, the results of this paper confirm the fact that GWP are sub-optimal insofar as they do not satisfactorily reflect to the ‘true’ social value of greenhouse gases. Furthermore, the analysis shows that –even under simplifying assumptions– changing the time-horizon in the computation is not sufficient to correct the bias induced by the use of GWP.

The main point made in this paper is that economists were not asked this question. For various reasons, the IPCC assessment reports have promoted the use of the GWP index in multi-greenhouse gas assessments, and the concept has been as successful as being included in the Kyoto Protocol itself. Economists thus have to deal with the concept, which will most likely remain as a key-feature in the on-going climate change negotiations. This paper proposes a general formulation of a second-best tax, which explicitly takes into account the equivalence rule as given. The tax is shown to be a combination of the 'true' marginal social values in all greenhouse gases (marginal social damage), weighted by the marginal costs of substituting more- by less-polluting goods (abatement costs) *and* the corresponding GWP. In general, this tax should not be constant, as it should reflect the change in marginal damage over time.

This work should be extended in several directions. We only mention two directions for further research. First, the properties of the second-best tax have to be further investigated from an empirical perspective. In particular, the system dynamics have to be compared under welfare-based tax and the second-best, GWP-based tax. Second the sensitivity of the system to change in key-parameters, such as the marginal damage or the temperature response to concentrations is still to be tested. The economic intuition is that the second-best, GWP based tax is less sensitive to changes in the value of marginal damage. This can be tested using the dynamic framework proposed in this paper, calibrated to the estimated climate-related parameters.

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